

8.44) a) VERIFY THAT THE FOLLOWING SOLVES THE RADIAL EQUATION FOR $n=2$ (8.107)

$$R_{2S}(r) = A \left(2 - \frac{r}{a_B} \right) e^{-\frac{r}{2a_B}}$$

b) FIND A FROM NORMALIZATION.

THE RADIAL EQUATION IS:

$$\frac{d^2}{dr^2}(rR) = \left[\frac{1}{a_B^2 n^2} - \frac{2}{a_B r} \right] (rR) \quad (8.107)$$

TO SHOW IT'S A SOLUTION, TAKE DERIVATIVES,

$$\frac{d}{dr}(rR) = A \frac{d}{dr} \left[\left(2r - \frac{r^2}{a_B} \right) e^{-\frac{r}{2a_B}} \right]$$

$$= A \left[\left(2 - \frac{2r}{a_B} \right) - \frac{1}{2a_B} \left(2r - \frac{r^2}{a_B} \right) \right] e^{-\frac{r}{2a_B}}$$

$$= A \left(2 - \frac{3r}{a_B} + \frac{r^2}{2a_B^2} \right) e^{-\frac{r}{2a_B}}$$

$$\frac{d^2}{dr^2}(rR) = A \frac{d}{dr} \left(2 - \frac{3r}{a_B} + \frac{r^2}{2a_B^2} \right) e^{-\frac{r}{2a_B}}$$

$$= A \left[\left(0 - \frac{3}{a_B} + \frac{2r}{2a_B^2} \right) - \frac{1}{2a_B} \left(2 - \frac{3r}{a_B} + \frac{r^2}{2a_B^2} \right) \right] e^{-\frac{r}{2a_B}}$$

$$= A \left[-\frac{4}{a_B} + \frac{5r}{2a_B^2} - \frac{r^2}{4a_B^3} \right] e^{-\frac{r}{2a_B}}$$

SUBSTITUTE INTO (8.107) \rightarrow For $n=2$

$$\cancel{A} \left[\frac{5r}{2a_B^2} - \frac{4}{a_B} - \frac{r^2}{4a_B^3} \right] \cancel{e^{-\frac{r}{2a_B}}} = \left[\frac{1}{4a_B^2} - \frac{2}{a_B r} \right] \cancel{A} r \left(2 - \frac{r}{a_B} \right) \cancel{e^{-\frac{r}{2a_B}}}$$

$$\frac{5r}{2a_B^2} - \frac{4}{a_B} - \frac{r^2}{4a_B^3} = \frac{r}{2a_B^2} - \frac{4r}{a_B} - \frac{r^2}{4a_B^3} + \frac{2r^2}{a_B^2 r}$$

$$\cancel{\frac{5r}{2a_B^2}} - \frac{4}{a_B} - \cancel{\frac{r^2}{4a_B^3}} = \frac{5r}{2a_B^2} - \frac{4}{a_B} - \frac{r^2}{4a_B^3} \rightarrow$$

8.44) CONTINUED

Thus

$$0 = 0$$

$$\text{AND } R_{25} = A \left(2 - \frac{r}{a_B} \right) e^{-\frac{r}{2a_B}} \quad \underline{\underline{\text{IS A SOLUTION!}}}$$

b) FIND A FROM NORMALIZATION

$$\Rightarrow \int_0^{\infty} P(r) dr = \int_0^{\infty} 4\pi r^2 |R(r)|^2 dr = 1$$

Thus

$$\int_0^{\infty} 4\pi r^2 \left[A^2 \left(2 - \frac{r}{a_B} \right)^2 e^{-\frac{r}{a_B}} \right] dr =$$

$$\Rightarrow = 4\pi A^2 \int_0^{\infty} r^2 \left(4 - \frac{4r}{a_B} + \frac{r^2}{a_B^2} \right) e^{-\frac{r}{a_B}} dr$$

$$= 4\pi A^2 \int_0^{\infty} \left[4r^2 e^{-\frac{r}{a_B}} - \frac{4r^3}{a_B} e^{-\frac{r}{a_B}} + \frac{r^4}{a_B^2} e^{-\frac{r}{a_B}} \right] dr$$

Use TZD II p 683 AND CRC # 369 p. 232

$$\int_0^{\infty} x^n e^{-\frac{x}{b}} dx = n! b^{(n+1)}, \quad b = a_B \text{ HERE}$$

Thus

$$\int_0^{\infty} P(r) dr = 4\pi A^2 \left[4 \overset{n=2}{(2)} (a_B)^3 - \frac{4}{a_B} \overset{n=3}{(6)} (a_B)^4 + \frac{1}{a_B^2} \overset{n=4}{(24)} (a_B)^5 \right]$$

$$= 4\pi A^2 \left[8a_B^3 - \cancel{24a_B^3} + \cancel{24a_B^3} \right]$$

$$= 32\pi A^2 a_B^3 = 1 \quad \leftarrow \text{TO NORMALIZE}$$

$$\Rightarrow \boxed{A = \frac{1}{\sqrt{32\pi a_B^3}}} \quad \Rightarrow \boxed{R_{25}(r) = \frac{1}{\sqrt{32\pi a_B^3}} \left(2 - \frac{r}{a_B} \right) e^{-\frac{r}{2a_B}}}$$

FULL R_{25} EXPRESSION.