8.43) THE PROBABILITY OF FINDING THE e^- IN $r > a$ IS

$$P(e^- \text{ IN } r>a) = \int_a^\infty P(r)\,dr.$$

FIND THE PROBABILITY OF FINDING A 1s e^- OUTSIDE a_B.

FOR a 1s e^-, $n=1$, $\ell=0$, AND $m=0$. TABLE LISTS $R_{1s}(r)$ AS

$$R_{1s}(r) = \frac{2}{\sqrt{a_B^3}}\, e^{-r/a_B}$$

WHERE THE CONSTANT INCLUDES THE CONSTANT FOR $\Theta(\theta)$.

FOLLOWING THE FOOTNOTE AFTER TABLE 8.2, THE PROBABILITY DENSITY IS

$$P(r) = r^2 |R(r)|^2 = \frac{4}{a_B^3} r^2 e^{-2r/a_B}$$

AND

$$P(e^- \text{ IN } r > a_B) = \frac{4}{a_B^3}\int_{a_B}^{\infty} r^2 e^{-2r/a_B}\,dr \qquad (1)$$

INTEGRATE BY PARTS

LET $u = r^2$ $\qquad dv = e^{-2r/a_B}\,dr$

$du = 2r\,dr$ $\qquad v = -\frac{a_B}{2} e^{-2r/a_B}$

THEN

$$\int_{a_B}^{\infty} r^2 e^{-2r/a_B}\,dr = -\frac{a_B}{2} r^2 e^{-2r/a_B}\Big|_{a_B}^{\infty} + a_B\int_{a_B}^{\infty} r e^{-2r/a_B}\,dr$$

$$= +\frac{a_B^3}{2} e^{-2} + a_B\int_{a_B}^{\infty} r e^{-2r/a_B}\,dr$$

LET $u = r$ $\qquad dv = e^{-2r/a_B}\,dr$

$dr = dr$ $\qquad v = -\frac{a_B}{2} e^{-2r/a_B}$

$$\int_{a_B}^{\infty} r^2 e^{-2r/a_B}\,dr = \frac{a_B^3}{2e^2} + a_B\left\{-\frac{a_B}{2} r e^{-2r/a_B}\Big|_{a_B}^{\infty} + \right.$$

$$\left. + \frac{a_B}{2}\int_{a_B}^{\infty} e^{-2r/a_B}\,dr \right\} \longrightarrow$$

8.43) CONTINUED

$$\int_{a_B}^{\infty} r^2 e^{-2r/a_B}\,dr = \frac{a_B^3}{2e^2} + \frac{a_B^3}{2e^2} + \frac{a_B^2}{2}\left(-\frac{a_B}{2}\, e^{-2r/a_B}\right|_{a_B}^{\infty}$$

$$= \frac{a_B^3}{2e^2} + \frac{a_B^3}{2e^2} + \frac{a_B^3}{4}\, e^{-2}$$

$$= \frac{5a_B^3}{4e^2}$$

THE PROBABILITY IS THUS [FROM (1)]

$$P(e^- \text{ IN } r > a_B) = \frac{\cancel{4}}{\cancel{a_B^3}}\left(\frac{5\cancel{a_B^3}}{\cancel{4}e^2}\right)$$

$$\boxed{P(e^- \text{ IN } r > a_B) = \frac{5}{e^2} \approx 0.68}$$

$\Rightarrow$ IT'S MORE LIKELY TO BE OUTSIDE a_B THAN INSIDE!

$\Rightarrow$ USE MATHEMATICA TO SHOW THE AREAS UNDER THE PROBABILITY DENSITY CURVES.