Homework Set 7: Drag
Due Friday, September 22, 2023

1) A particle of mass m slides down an inclined plane under the influence of gravity. If the motion is resisted by a force f = kmv^2, show that the time required to move a distance d after starting from rest is

$$t = \frac{\cosh^{-1}(e^{kd})}{\sqrt{kg\sin\theta}}$$

where θ is the angle of inclination of the plane. Hints: You may find the $\Sigma F = mv(dv/dx)$ form of NSL helpful. Take the derivative of $\cosh^{-1}(e^{kx})$ to find the necessary form of the integrand of the integral over x after you've found v(t) and substituted dx/dt for it. Go back and work on the "Remembering Math" homework (set 0). Also, m in the drag force is for algebraic convenience, drag does ***not*** depend on the mass of the object.

2) A gun is fired straight up. Assuming that the air drag on the bullet varies quadratically with speed, derive the equations describing how velocity varies with height (Hint: You must solve NSL for upward and downward motion separately. Label the variables as up or down and use limits (0 to t, $y_{0,up}$ to y_{up}, $y_{0,down}$ to y_{down}).)

$$v_{up}^2 = Ae^{-2ky_{up}} - \frac{g}{k}, \quad \text{where} \quad A = \frac{g + kv_{0,up}^2}{k} \qquad \text{(upward motion)}$$

$$v_{down}^2 = \frac{g}{k} - Be^{2ky_{down}}, \quad \text{where} \quad B = \left(\frac{g - kv_{0,down}^2}{k}\right)e^{-2ky_{0,down}} \qquad \text{(downward motion)}$$

in which A and B are constants of integration, g is the acceleration of gravity, k is the drag constant [$F_{Drag} = mkv^2$], and m is the mass of the bullet NOTE: y is measured positive upward and m in the drag force is for algebraic convenience, drag does ***not*** depend on the mass of the object.

Use these to show that when the bullet hits the ground on its return, its speed will be

$$v = \frac{v_o v_t}{\sqrt{v_o^2 + v_t^2}}$$

in which v_o is the initial upward speed and

$$v_t = \sqrt{\frac{g}{k}} = \text{terminal speed}$$

HINT: Note that the final height for the upward motion, $y_{up} = y_{top}$, becomes $y_{0,down}$ for the downward motion. Find an expression for y_{top} and use this to find the velocity (v_{down}) when the bullet returns to the ground, $y_{down} = 0$