Homework Set 9: Equilibrium & Stability
Due Friday, October 4, 2024

Problems From TM5.

1) 2-43 A particle is under the influence of a force $F = -kx + kx^3/\alpha^2$. Where k and α are constants and k is positive. Determine U(x) and discuss the motion using computer plots[1] of F(x) and U(x). What is the situation and what will occur when $E = (1/4)k\alpha^2$? (Determine the stability at this energy analytically, don't just look at the plots.)

2) 2-47 Consider a particle moving in the region x > 0 under the influence of the potential

$$U(x) = U_0\left(\frac{\alpha}{x} + \frac{x}{\alpha}\right)$$

where U_0 = 1 J and α = 2 m. Plot[1] the potential (using a computer), find the equilibrium points, and determine whether they are maxima or minima (Do this analytically, don't just look at the plots!).

Calculus and Algebra Practice

3) Show and explain all the steps to get from setting

$$\left.\frac{dU(x_1)}{dx_1}\right|_{x_1=x_0} = \frac{d}{dx_1}\left\{-m_1 g x_1 - m_2 g\sqrt{\left(\frac{b-x_1}{2}\right)^2 - d^2}\right\} = 0$$

taking the derivative, and simplifying to

$$x_0 = b - \frac{4m_1 d}{\sqrt{4m_1^2 - m_2^2}}$$

(The point is for you to *explain* to someone who doesn't know how to do it how it's done, not show me you can do it)

4) Show all the steps to get from

$$\left.\frac{d^2U(x_1)}{dx_1^2}\right|_{x_1=x_0} = \frac{d}{dx_1}\left\{-m_1 g + \frac{m_2 g(b-x_1)}{4\sqrt{\left(\frac{b-x_1}{2}\right)^2 - d^2}}\right\}$$

to

$$\left.\frac{d^2U(x_1)}{dx_1^2}\right|_{x_1=x_0} = \frac{g\left(4m_1^2 - m_2^2\right)^{3/2}}{4m_2^2 d}$$

by taking the derivative & simplifying, then substituting the value of x_1 from above and simplifying again. I recommend you not substitute for x_1 at the beginning … it just makes it too complicated.

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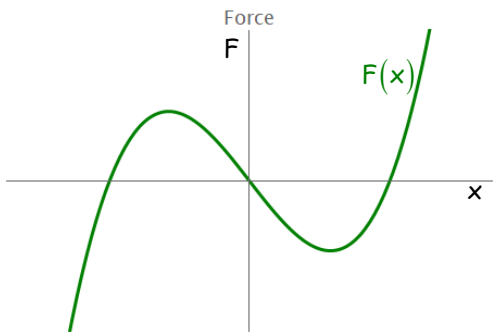
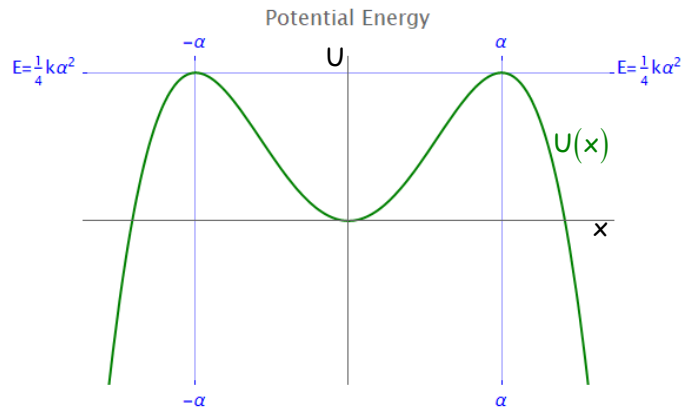
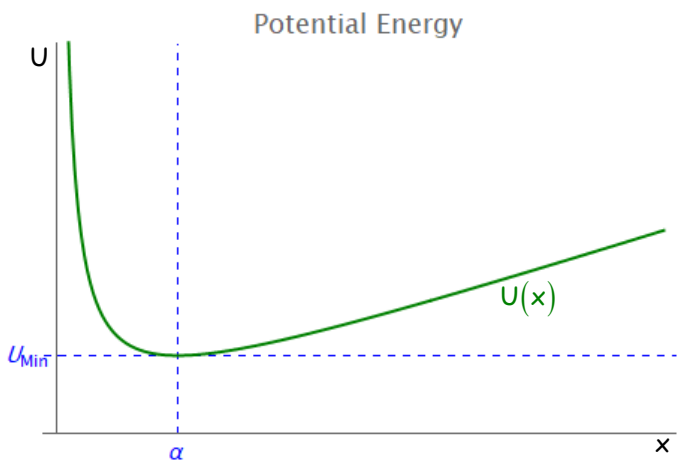
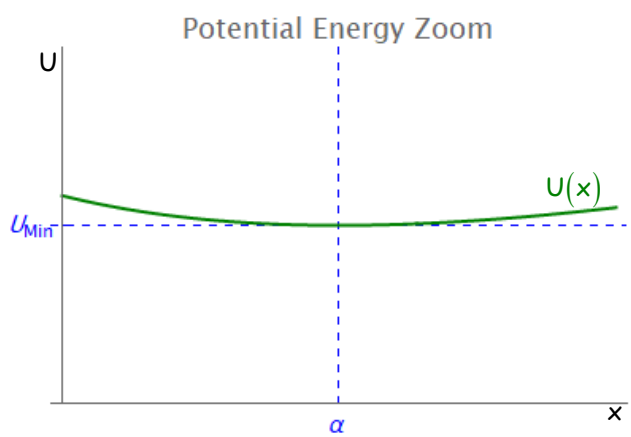
[1] Mathematica plots for both problems are on the back. All of you should be learning Python or some other plotting program (Desmos will do, but it's not great). If you plot it yourself, I'll give you up to 5 points extra credit. Use my plots as examples of how to lable them and indicate important values.

Plots of F(x) and U(x) for TM5 problem 2.43





Plot of U(x) for TM5 problem 2.47



Zooming in on the minimum.