Homework Set 11: Simple & Damped Harmonic Motion
Due Wednesday, October 11, 2023

Problems From TM5.

1) *3-2* Allow the motion of a *100 g* mass attached to a spring with a force constant of $k = 10^4$ *dyne/cm* initially displaced *3 cm* from the equilibrium point and released from rest, to take place in a resisting medium. After oscillating for *10 s*, the maximum amplitude decreases to half the initial value. Calculate

a) the damping parameter β, and

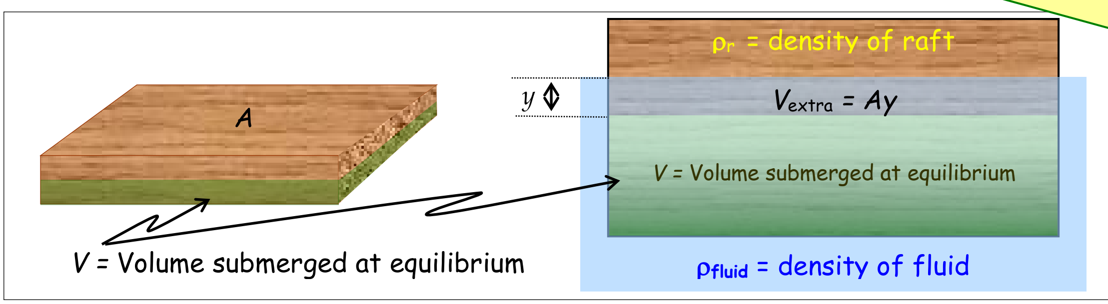
b) the frequency ν_S (compare with the undamped frequency ν_N (these are f_S and f_N).

1 dyne = 1 g-cm/s^2 = 10^{-5} N but don't convert! **Stay in cgs!**

2) *3-7.* A raft of uniform cross-sectional area *A* and of mass density ρ_r floats in a liquid of density ρ_{fluid}. At equilibrium it displaces a volume *V*. Show that the period of small oscillations about the equilibrium position [due to buoyancy: $F_{buoyancy}$ = Weight of fluid displaced = (mass of fluid)g = (ρ_{fluid}V)g] is given by

$$\tau = 2\pi\sqrt{\frac{V}{gA}}$$

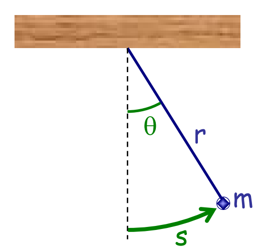
This is not hard … write out NSL for equilibrium to find the mass of the raft then for the buoyancy as the restoring force: $-F_{restore} = m\ddot{y}$.



3) *3-10* If the amplitude of a damped oscillator decreases to *1/e* of its initial value after *n* periods, show that the ratio of the frequency of the oscillator to that of the corresponding undamped oscillator must be approximately

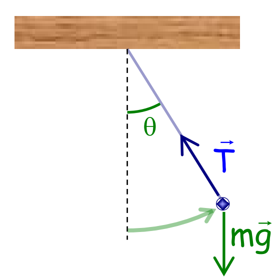
$$\frac{\omega_S}{\omega_N} \cong 1 - \frac{1}{8\pi^2 n^2}$$

(To get this expression, derive the exact expression $\frac{\omega_S}{\omega_N} = \left(1 + \frac{1}{4\pi^2 n^2}\right)^{-\frac{1}{2}}$ and expand it)



4) *3-12* A simple pendulum consists of a mass m suspended from a fixed point by a weightless, extensionless rod of length *r*. Obtain the equation of motion and, in the approximation that *sin* $\theta \approx \theta$, show that the natural frequency is $\omega_N = \sqrt{g/r}$ where *g* is the gravitational field strength. Discuss the motion in the event that the motion takes place in a viscous medium with retarding force, $F_{Drag} = 2m\sqrt{gr}\,\dot{\theta}$.

(Find β and compare it to ω_N to determine which damping solution to use … underdampd or critically damped.)



Don't neglect the FBDs!
What is $F_{Restore}$?
Where is F_{Drag}?