# HOMEWORK SET 12: DAMPED AND DRIVEN HARMONIC MOTION
## Due Monday, October 16, 2023

## PROBLEMS FROM TM5.

**1)** *3-2* Altered: Allow the motion of a *100 g* mass attached to a spring with a force constant of $k = 10^4$ *dyne/cm* initially displaced *3 cm* from the equilibrium point and released from rest, to take place in a resisting medium. After oscillating for 10 $T_S$ (10 system periods), the maximum amplitude decreases to half the initial value. Calculate

- **a)** the damping parameter $\beta$, and
- **b)** the frequency $\nu_s$ (compare with the undamped frequency $\nu_N$ (these are $f_S$ and $f_N$).

1 dyne = 1 g-cm/s$^2$ = $10^{-5}$ N but don't convert! **Stay in cgs!**

**1)** *3-11* Derive the expressions (*by hand … show the math*) for the energy and energy-loss curves shown in Figure 3-8 for the damped oscillator and reproduce them using your favorite plotting program.
[Make them look like those in the text! MATCH the algebraic expressions from TM5:

$$E(t) = \frac{mA^2}{2} e^{-2\beta t}\left\{\omega_N^2 + \beta^2 \cos 2\left(\omega_S t - \delta\right) + \beta\sqrt{\omega_N^2 - \beta^2}\,\sin 2\left(\omega_S t - \delta\right)\right\}$$

$$\frac{dE(t)}{dt} = mA^2 e^{-2\beta t}\left\{\beta\left(\omega_S^2 - \beta^2\right)\cos\left[2\left(\omega_S t - \delta\right)\right] - 2\beta^2\omega_S \sin\left[2\left(\omega_S t - \delta\right)\right] - \beta\omega_N^2\right\}$$

DON'T USE NUMBERED SUBSCRIPTS IN MATHEMATICA! Use $\omega_N$ for $\omega_0$ (for the natural frequency) and $\omega_S$ for $\omega_1$ (for the system frequency … $\omega_D$ will be for the driving frequency). The values I used for light damping were $\omega_N = 1$, $\beta = 0.1$, $m = 2$, $A = 1$, and $\delta = 0$.]

**2)** *3-16* Discuss the motion of a particle described by equation 3.34 in the event that b < 0 (i.e., the damping is negative).

**3)** *3-24* For $\beta = 0.2\ s^{-1}$, produce computer plots like those shown in Figure 3-15 for a sinusoidal driven, damped oscillator where $x_p(t)$, $x_c(t)$, and the sum $x(t)$ are shown. Let $k = 1\ kg/s^2$ and $m = 1\ kg$. Do this for values of $\omega_D/\omega_S$ of 1/9, 1/3, 1.1, 3 and 6. For the $x_c(t)$ solution (Eqn. 3.40), let the phase angle $\delta = 0$. And the amplitude $A = -1\ m$. For the $x_p(t)$ solution (Eqn. 3.60), let $F_0/m = 1\ m/s^2$, but calculate $\delta$. What do you observe about the relative amplitudes of the two solutions as $\omega_D$ increases? Why does this occur? For $\omega_D/\omega_S = 6$, let $F_0 = 20\ m/s^2$ for $x_p(t)$ and produce the plot again.

You may use any plotting program you are familiar with but you must use a computer. The point is for you to observe how the transient, steady state, and sum functions are affected by the ratio of frequencies. Make sure to explain and label your plots … show which curves are the transient, the steady state, and the sum (it's best to use colors). I've plotted them using Mathematica. You may use my file for problem 3-25 (TM5Pr3_25) on T:/O'Donoghue/Mechanics/Mathematica as a template. Since 3-25 is critically damped and 3-24 is underdamped, if you use 3-24, make sure you teach it the underdamped $x_c(t)$ for 3-24 (kill the kernel if you've been running 3-25)! If you don't, your transient will be incorrect for 3-25.

# ENERGY AND ENERGY LOSS BY THE DAMPED HARMONIC OSCILLATOR
## TM5[1] CHAPTER 3



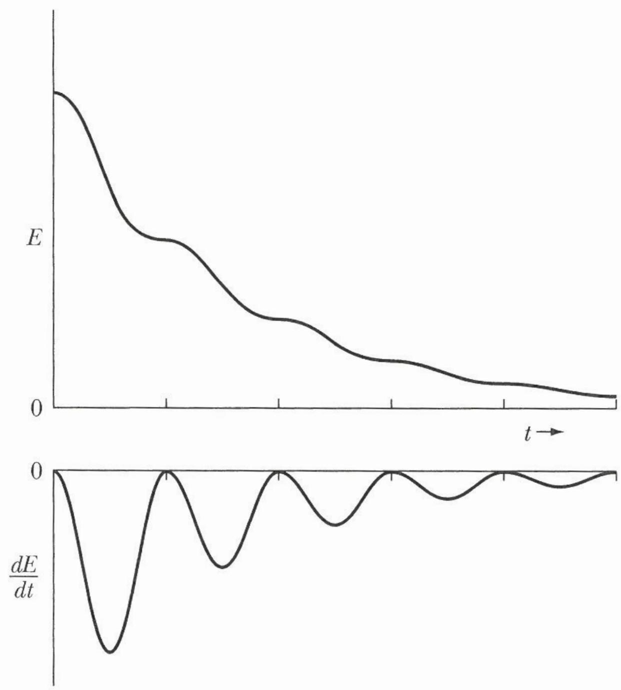


**FIGURE 3-8** The total energy and rate of energy loss for the damped oscillator.

[1] Thornton, T.T. and Marion, J. B., (2004). Classical Dynamics of Particles and Systems. 5th Ed. Belmont, CA: Brooks-Cole.