HOMEWORK SET 13: DRIVEN HARMONIC MOTION
Due Friday, October 20, 2023

PROBLEMS FROM TM5.

1) 3-24 *Altered* For β = 0.2 s^{-1}, Mathematic plots like those shown in Figure 3-15 for a sinusoidal driven, damped oscillator where $x_p(t)$, $x_c(t)$, and the sum x (t) are displayed on the back of this sheet. To produce them, I let k = 1 kg/s^2, m = 1 kg, A = -1 m, the phase angle δ = 0, and plotted values of ω_D/ω_S of 1/9, 1/3, 1.1, 3 and 6. For the $x_p(t)$ solution (Eqn. 3.60), I let F_0/m = 1 m/s^2, but calculatde δ. For the last plot, in the $x_p(t)$ solution (Eqn. 3.60), I let F_0/m = 20 m/s^2.

What do you observe about the relative amplitudes of the two solutions as ω_D increases? Why does this occur? For ω_D/ω_S = 6, let F_0 = 20 m/s^2 for $x_p(t)$ and produce the plot again.

TM5[1] CHAPTER 3

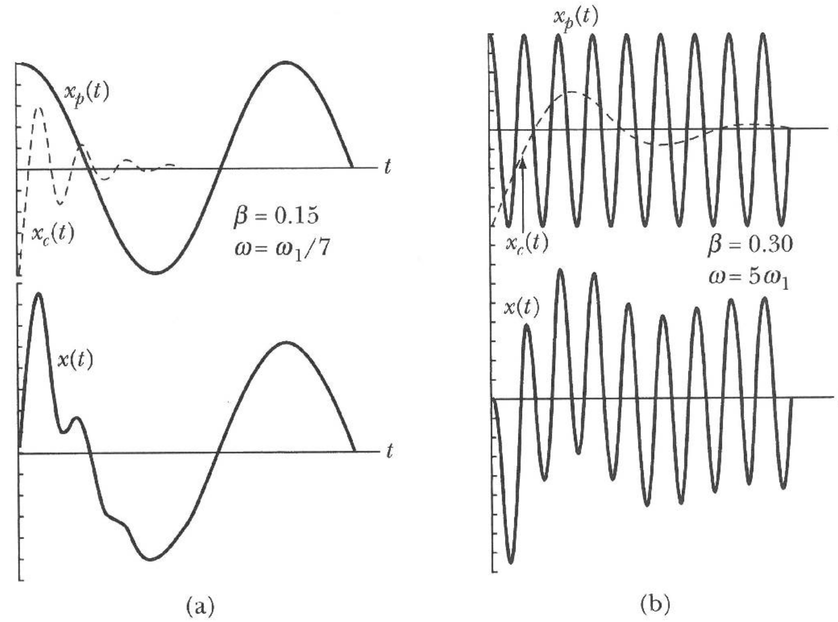

FIGURE 3-15 Examples of sinusoidal driven oscillatory motion with damping. The steady-state solution x_p, transient solution x_c, and sum x are shown in (a) for driving frequency ω greater than the damping frequency $\omega_1 (\omega > \omega_1)$ and in (b) for $\omega < \omega_1$.

[1] Thornton, T.T. and Marion, J. B., (2004). Classical Dynamics of Particles and Systems. 5th Ed. Belmont, CA: Brooks-Cole.

The plots show driven, under dampled harmonic oscillations for

$$x(t) = Ae^{-\beta t}\cos(\omega_S t) + \frac{F_0/m}{\sqrt{\left(\omega_N^2-\omega_D^2\right)^2+4\beta^2\omega_D^2}}\cos(\omega_D t-\delta), \text{ where } \delta = \tan^{-1}\left(\frac{2\beta\omega_D}{\omega_N^2-\omega_D^2}\right)$$

with $F_0/m = k = 1$, $\delta_{transient} = 0$, $A = -1$, and $\beta = 0.2\ s^{-1}$ (giving $\omega_N = 1\ s^{-1}$ and $\omega_S = 0.9798\ s^{-1}$) and ω_D as multiples of ω_N as given on each plot.