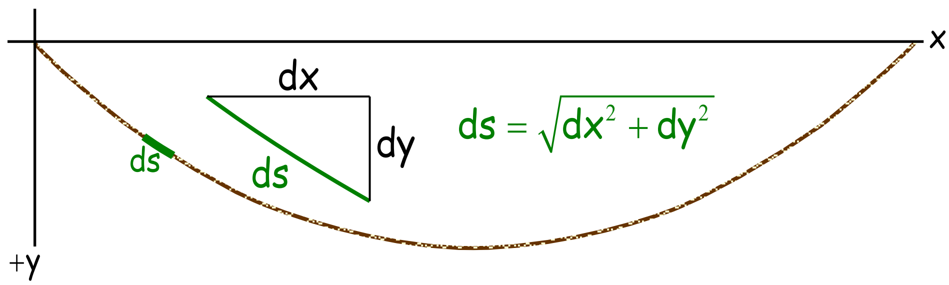
HOMEWORK SET 15: EULER'S EQUATION: EXTRA CREDIT
Due Monay, November 8, 2021

1)[20] Derive the catenary curve using Euler's Equation (*SHOW ALL THE MATH STEPS!*)



$$dU = gydm = gy(\lambda ds) = g\lambda yds$$

$$U = g\lambda \int yds$$
$$= g\lambda \int y\sqrt{dx^2 + dy^2}$$

Minimize the functional of the potenial energy using Euler's equation

$$U = g\lambda \int y\sqrt{1+(y')^2}\,dx \qquad \frac{\partial f}{\partial y} - \frac{d}{dx}\frac{\partial f}{\partial y'} = 0$$

After taking derivatives and substituting into Euler's equation, you should get $1+(y')^2 - yy'' = 0$. Multiply through by y' and rarrange to show this is the diffeence of the derivatives of $\ln(y)$ and $\ln\left[(y')^2+1\right]$.

$$\frac{y'}{y} - \frac{y'y''}{1+(y')^2} = 0 .$$

Integrate these over indefinite limits and write the constant(s) as $\ln(A)$ to derive $A^2y^2 - 1 = (y')^2$.

This can then be written as a separable DE. If you recognize the derivative of the inverse cosh and your freedom in writing constants in any form, you can get the equation of the catenary curve shown. Plot this with your favorite software to show it does give the catenary.

$$\frac{dy}{dx} = \frac{1}{\sqrt{A^2y^2-1}} \quad \Rightarrow \quad y(x) = C\cosh(Ax) + B$$