

HOMEWORK SET 17: THE HAMILTONIAN

Due Friday, November 18, 2022

PROBLEMS FROM TM5

- 1) 7-26 Determine the Hamiltonian and Hamilton's equations of motion for
- a simple pendulum and
 - a simple Atwood machine (single pulley)

HINTS: Write H in terms of p , not \dot{q} !

- 2) 7-24 Consider a simple plane pendulum consisting of a mass m attached to a string of length ℓ . After the pendulum is set into motion, the length of the string is shortened at a constant rate

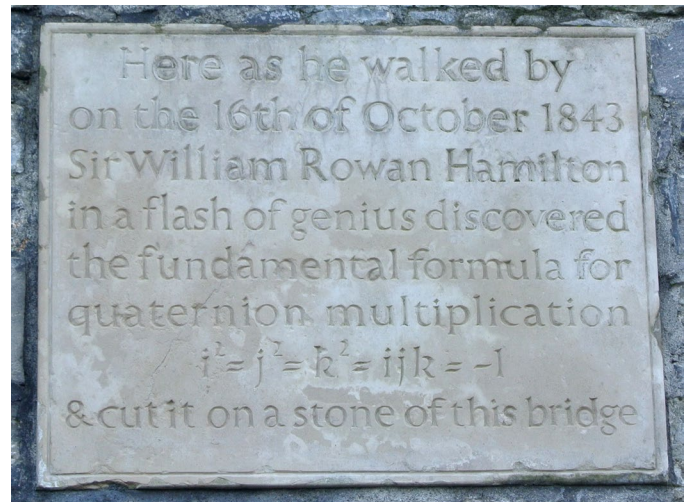
$$\frac{d\ell}{dt} = \dot{\ell} = -\alpha = \text{constant}$$

The suspension point remains fixed. Write the Lagrangian and use the definition below to determine the Hamiltonian. Compare the Hamiltonian to the total energy and discuss the conservation of energy for the system. HINTS: The definition of the Hamiltonian is

$$H = \sum_j \dot{q}_j \frac{\partial L}{\partial \dot{q}_j} - L = \sum_j \dot{q}_j p_j - L$$

The changing ℓ adds an element $(\vec{v})^2 = \vec{v} \cdot \vec{v}$ to the kinetic energy. Recall that and think about the directions of $\dot{\ell}$ and $\dot{\theta}$. You should get

$$L = \frac{1}{2}m(\ell^2\dot{\theta}^2 + \alpha^2) + mg\ell \cos\theta \qquad \text{and} \qquad H = \frac{p_\theta^2}{2m\ell^2} - \frac{1}{2}m\alpha^2 - mg\ell \cos\theta$$



<https://en.wikipedia.org/wiki/Quaternion>

POSTAGE STAMPS OF
IRELAND
(ÉIRE)

EUROPA
1983

Newgrange Winter Solstice

Hamilton Equations

Neolithic patterns incised on stone at Newgrange and William Rowan Hamilton's discovery in 1843 of the mathematical equations quaternions are honored on these Europa stamps issued on May 4, 1983. During the Winter Solstice (December 21) the rays of the rising sun penetrate a passage in the neolithic tomb and illuminate the burial chamber deep inside the mound.