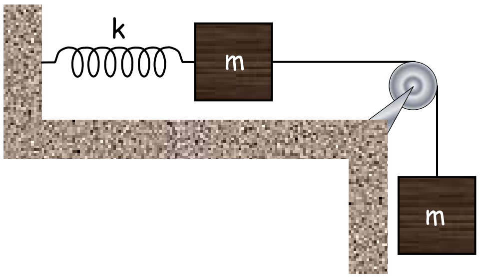
HOMEWORK SET 17: FUN WITH LAGRANGE
Due Wednesday, November 20, 2013



PROBLEM FROM BARGER & OLSSON

1) *3-2* Two equal masses are constrained by the spring-and-pulley system shown. Assume a massless pulley and a frictionless surface. Let x be the extension of the spring from its relaxed length. Derive the equations of motion by Lagrangian methods. Solve for x as a function of time with the boundary conditions $x(t=0)=\dot{x}(t=0)=0$. Answer: $x(t)=\frac{mg}{k}\left(1-\cos\left(\sqrt{\frac{k}{2m}}t\right)\right)$

You should give you a differential equation, $\ddot{x}+\frac{k}{2m}x=\frac{g}{2}$ for which you should guess a solution $x=A+B\cos(\omega_N t)$.

PROBLEMS FROM TM5

2) *7-2* Work out Example 7.6 showing all the steps, in particular those leading to Equations 7.36 and 7.41. Explain why the sign of the acceleration a cannot affect the frequency ω. Give an argument why the signs of a^2 and g^2 in the solution of ω^2 in Equation 7.42 are the same.

Write down *every step* in taking the derivatives of the Lagrangian … no short-cuts or you'll miss something! Be carful about the difference between partial and full derivatives!

3) *7-3* A sphere of radius ρ is constrained to roll without slipping on the lower half of the inner surface of a hollow cylinder of inside radius R. Here, θ is the angle between the vertical and the position of the center of mass of the small sphere and φ is the angle through which the small sphere has rolled. Determine the Lagrangian function, the equation of constraint and show that Lagrange's equations of motion, for the coordinates shown, are

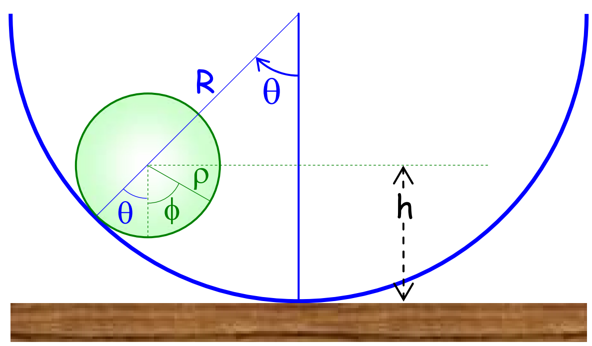
$$-(R-\rho)\,mg\sin\theta-m(R-\rho)^2\ddot{\theta}+\lambda(R-\rho)=0$$
$$-\tfrac{2}{5}m\rho^2\ddot{\phi}-\lambda\rho=0,$$

where

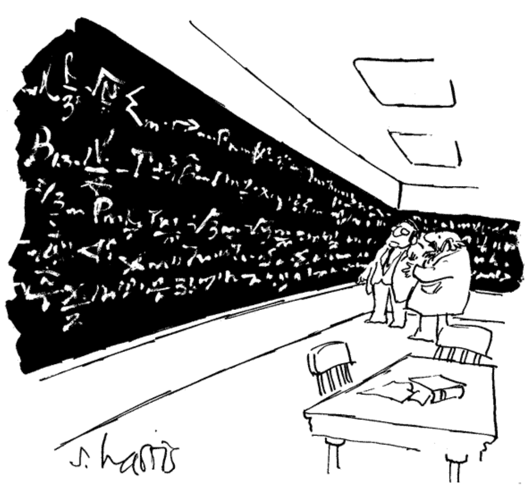
$$\lambda=-\tfrac{2}{5}m(R-\rho)\ddot{\theta}=-\tfrac{2}{5}m\rho\ddot{\phi}$$

Also show that the frequency of small oscillations is

$$\omega=\sqrt{\frac{5g}{7(R-\rho)}}.$$



The energies are due to the height of the CM of the sphere and its translational ($\frac{1}{2}mv^2$) and rotational ($\frac{1}{2}I\omega^2=\frac{1}{2}I\dot{\phi}^2$) motions. The rolling constraint is that the distance moved by the center of mass is equal to the arc length along the sphere: $s_{cm}=r_{sphere}\phi_{sphere}$. Describe s_{cm} in terms of θ and R-ρ and use Lagrange's equation with undetermined multipliers (TM5 Equation 7.65).



"But this is just a simplistic way of looking at the problem."