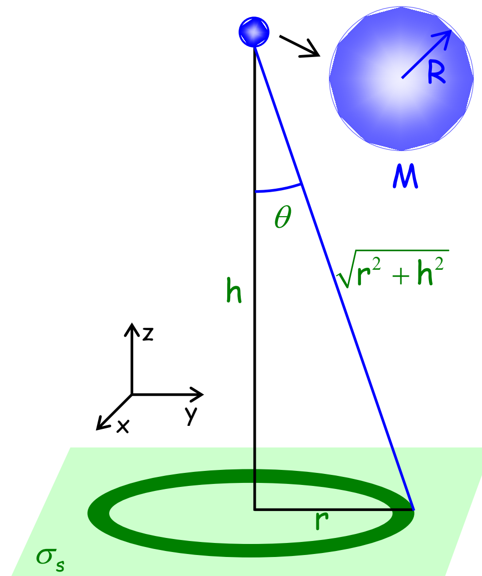
HOMEWORK SET 18: GRAVITATIONAL FORCE & POTENTIAL
Due Monday, November 29, 2021

1) *5-3* Assuming that air resistance is unimportant, derive an expression for the minimum velocity a particle must have at the surface of Earth to escape from Earth's gravitational field. Obtain a numerical value for the result. (This is called the *escape velocity*) Use energy with $(T + U)_{surface} = (T + U)_{\infty} = 0$! You should get v_e = 11.2 km/s (25,000 mph)

2) *5-16* A uniformly solid sphere of mass *M* and radius *R* is fixed a distance *h* above a thin infinite sheet of mass density σ (mass/area). With what force does the sphere attract the sheet?

Find the force of the plane on the sphere then take its opposite (Newton's 3rd Law) as the force of the sphere on the plane. Take dm as that of a ring of radius r, width dr and length $2\pi r$ and integrate from r = 0 to ∞.

$$F_{z,\text{sphere on plane}} = +2\pi\sigma_s GM$$



3) *5-5* A particle falls to the Earth starting from rest at a great height. Neglect air resistance and show that the particle requires approximately 9/11 of the total time of fall to traverse the first half of the distance.

$$\frac{9}{11} \approx \frac{\left(\frac{\pi}{2}+1\right)}{\pi}$$

Start with NSL with (dp/dt) = mv(dv/dy). After the first round you should end up with an ugly integral as shown … with the substitution that will help you solve it (ultimately you'll find the integral of $\cos^2\theta$). It will be helpful to factor a $\sqrt{(1/y)}$ out of the denominator to leave $1 - (y/y_0)$ under the square root. Integrate using the appropriate limits (what is θ when $y = y_0$ and y = y?). Evaluate your answer for t_{total} and t_{half} then find the ratio.

$$\int_{y_{top}}^{y} \frac{dy}{\sqrt{2GM\left(\frac{1}{y}-\frac{1}{y}\right)}}$$

Start the substitution as shown. Substitute (cleverly) for the expression in the denominator, the limits, and don't forget dy!

$$\sqrt{y} = \sqrt{y_0}\cos\theta$$