Homework Set 18: The Hamiltonian
Due Friday, November 17, 2023

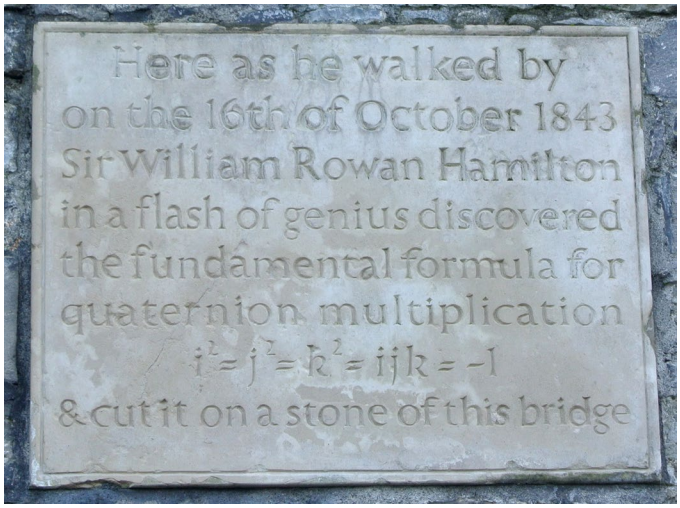
Problems From TM5

1) *7-26* Determine the Hamiltonian and Hamilton's equations of motion for

a) a simple pendulum and

b) a simple Atwood machine (single pulley)

HINTS: Write H in terms of p, not $\dot{q}$!



https://en.wikipedia.org/wiki/Quaternion

2) *7-24* Consider a simple plane pendulum consisting of a mass *m* attached to a string of length ℓ. After the pendulum is set into motion, the length of the string is shortened at a constant rate

$$\frac{d\ell}{dt} = \dot{\ell} = -\alpha = \text{constant}$$

The suspension point remains fixed. Write the Lagrangian and use the definition below to determine the Hamiltonian. Compare the Hamiltonian to the total energy and discuss the conservation of energy for the system. HINTS: The definition of the Hamiltonian is

$$H = \sum_j \dot{q}_j \frac{\partial L}{\partial \dot{q}_j} - L = \sum_j \dot{q}_j p_j - L$$

The changing ℓ adds an element $(\vec{v})^2 = \vec{v} \bullet \vec{v}$ to the kinetic energy. Recall that and think about the directions of $\dot{\ell}$ and $\dot{\theta}$. You should get and

$$L = \frac{1}{2}m\left(\ell^2\dot{\theta}^2 + \alpha^2\right) + mg\ell\cos\theta \qquad H = \frac{p_\theta^2}{2m\ell^2} - \frac{1}{2}m\alpha^2 - mg\ell\cos\theta$$