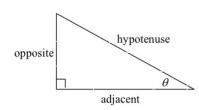
# Trig Cheat Sheet

## **Definition of the Trig Functions**

## Right triangle definition

For this definition we assume that  $0 < \theta < \frac{\pi}{2}$  or  $0^{\circ} < \theta < 90^{\circ}$ .



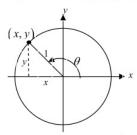
$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \csc \theta = \frac{\text{hypotenuse}}{\text{opposite}}$$

$$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \sec \theta = \frac{\text{hypotenuse}}{\text{adjacent}}$$

$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}} \qquad \cot \theta = \frac{\text{adjacent}}{\text{opposite}}$$

## Unit circle definition

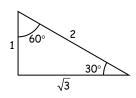
For this definition  $\theta$  is any angle.



$$\sin \theta = \frac{y}{1} = y \qquad \csc \theta = \frac{1}{y}$$

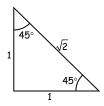
$$\cos \theta = \frac{x}{1} = x \qquad \sec \theta = \frac{1}{x}$$

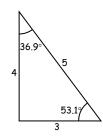
$$\tan \theta = \frac{y}{x} \qquad \cot \theta = \frac{x}{y}$$



$$\sin(30^\circ) = \frac{1}{2} \qquad \cos(30^\circ) = \frac{\sqrt{3}}{2} \qquad \tan(30^\circ) = \frac{1}{\sqrt{3}}$$

$$\sin(60^\circ) = \frac{\sqrt{3}}{2} \qquad \cos(60^\circ) = \frac{1}{2} \qquad \tan(60^\circ) = \sqrt{3}$$





$$Sin \left(45^{\circ}\right) = \frac{1}{\sqrt{2}} \quad Cos \left(45^{\circ}\right) = \frac{1}{\sqrt{2}} \quad Tan \left(45^{\circ}\right) = 1$$



Carpenter's 3-4-5 triangle for confirming a right angle. Measure & mark 4 and 3 feet (meters) out from a corner. Measure between the marks ... when it's 5, you have a right angle.

#### © 2005 Paul Dawkins

### Formulas and Identities

## **Tangent and Cotangent Identities**

$$\tan \theta = \frac{\sin \theta}{\cos \theta} \qquad \cot \theta = \frac{\cos \theta}{\sin \theta}$$

## **Reciprocal Identities**

$$csc \theta = \frac{1}{\sin \theta} \qquad sin \theta = \frac{1}{\csc \theta} 
sec \theta = \frac{1}{\cos \theta} \qquad cos \theta = \frac{1}{\sec \theta} 
cot \theta = \frac{1}{\tan \theta} \qquad tan \theta = \frac{1}{\cot \theta}$$

## **Pythagorean Identities**

$$\sin^2 \theta + \cos^2 \theta = 1$$
$$\tan^2 \theta + 1 = \sec^2 \theta$$
$$1 + \cot^2 \theta = \csc^2 \theta$$

### Even/Odd Formulas

$$\sin(-\theta) = -\sin\theta$$
  $\csc(-\theta) = -\csc\theta$   
 $\cos(-\theta) = \cos\theta$   $\sec(-\theta) = \sec\theta$   
 $\tan(-\theta) = -\tan\theta$   $\cot(-\theta) = -\cot\theta$ 

## Periodic Formulas

If *n* is an integer.  $\sin(\theta + 2\pi n) = \sin\theta \quad \csc(\theta + 2\pi n) = \csc\theta$   $\cos(\theta + 2\pi n) = \cos\theta \quad \sec(\theta + 2\pi n) = \sec\theta$   $\tan(\theta + \pi n) = \tan\theta \quad \cot(\theta + \pi n) = \cot\theta$ 

## Double Angle Formulas

$$\sin(2\theta) = 2\sin\theta\cos\theta$$

$$\cos(2\theta) = \cos^2\theta - \sin^2\theta$$

$$= 2\cos^2\theta - 1$$

$$= 1 - 2\sin^2\theta$$

$$\tan(2\theta) = \frac{2\tan\theta}{1 - \tan^2\theta}$$

# Degrees to Radians Formulas

If x is an angle in degrees and t is an angle in radians then

$$\frac{\pi}{180} = \frac{t}{x}$$
  $\Rightarrow$   $t = \frac{\pi x}{180}$  and  $x = \frac{180t}{\pi}$ 

## Half Angle Formulas

$$\sin^2 \theta = \frac{1}{2} (1 - \cos(2\theta))$$
$$\cos^2 \theta = \frac{1}{2} (1 + \cos(2\theta))$$

# $\tan^2 \theta = \frac{1 - \cos(2\theta)}{1 + \cos(2\theta)}$

Sum and Difference Formulas  

$$\sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta$$
  
 $\cos(\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta$   
 $\tan(\alpha \pm \beta) = \frac{\tan \alpha \pm \tan \beta}{1 \mp \tan \alpha \tan \beta}$ 

## Product to Sum Formulas

$$\sin \alpha \sin \beta = \frac{1}{2} \Big[ \cos(\alpha - \beta) - \cos(\alpha + \beta) \Big]$$

$$\cos \alpha \cos \beta = \frac{1}{2} \Big[ \cos(\alpha - \beta) + \cos(\alpha + \beta) \Big]$$

$$\sin \alpha \cos \beta = \frac{1}{2} \Big[ \sin(\alpha + \beta) + \sin(\alpha - \beta) \Big]$$

$$\cos \alpha \sin \beta = \frac{1}{2} \Big[ \sin(\alpha + \beta) - \sin(\alpha - \beta) \Big]$$

#### Sum to Product Formulas

$$\sin \alpha + \sin \beta = 2 \sin \left(\frac{\alpha + \beta}{2}\right) \cos \left(\frac{\alpha - \beta}{2}\right)$$

$$\sin \alpha - \sin \beta = 2 \cos \left(\frac{\alpha + \beta}{2}\right) \sin \left(\frac{\alpha - \beta}{2}\right)$$

$$\cos \alpha + \cos \beta = 2 \cos \left(\frac{\alpha + \beta}{2}\right) \cos \left(\frac{\alpha - \beta}{2}\right)$$

$$\cos \alpha - \cos \beta = -2 \sin \left(\frac{\alpha + \beta}{2}\right) \sin \left(\frac{\alpha - \beta}{2}\right)$$

#### **Cofunction Formulas**

$$\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta \qquad \cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$$

$$\csc\left(\frac{\pi}{2} - \theta\right) = \sec\theta \qquad \sec\left(\frac{\pi}{2} - \theta\right) = \csc\theta$$

$$\tan\left(\frac{\pi}{2} - \theta\right) = \cot\theta \qquad \cot\left(\frac{\pi}{2} - \theta\right) = \tan\theta$$

