TOPICS

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CALCULUS OF VARIATIONS

THE QUANTITY J EXPRESSED AS AN INTEGRAL

$$J = \int_{x_1}^{x_2} f\{y(x), y'(x); x\}\, dx$$

HAS EXTREMA WHERE

$$\frac{\partial f}{\partial y} - \frac{d}{dx}\frac{\partial f}{\partial y'} = 0 \qquad \text{EULER'S EQUATION}$$

HAMILTONIAN MECHANICS

THE LAGRANGIAN

$$L(q_i, \dot{q}_i) \equiv T(\dot{q}_i) - U(q_i)$$

HAMILTON'S PRINCIPLE STATES NATURE MINIMIZES THE TIME INTEGRAL OF THIS

$$\delta \int_{t_1}^{t_2} L(q_i, \dot{q}_i, t)\, dt = 0 \Rightarrow \frac{\partial L}{\partial q_i} - \frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i} = 0$$

FOR k EQUATIONS OF CONSTRAINT, $f_k(q_i, \dot{q}_i, t) = 0$

LAGRANGE'S EQUATIONS WITH UNDETERMINED MULTIPLIERS

$$\boxed{\frac{\partial L}{\partial q_i} - \frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i} + \sum_k \lambda_k(t)\frac{\partial f_k}{\partial q_i} = 0}$$

$$Q_i = \sum_k \lambda_k \frac{\partial f}{\partial q_i} \qquad \text{GENERALIZED FORCE}$$