TOPICS

- HARMONIC OSCILLATIONS
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  - MECHANICAL - ELECTRICAL EQUIVALENTS

SHM - SIMPLE HARMONIC MOTION

LINEAR RESTORING FORCE $F_{RESTORE} = -kx$

NO DAMPING

NSL: $\sum F = m\ddot{x}$

$-kx = m\ddot{x}$

DE: $\ddot{x} + \omega_N^2 x = 0$, $\omega_N^2 = \frac{k}{m}$

SOLUTION: $x = A\cos(\omega_N t - \delta)$

INITIAL CONDITIONS GIVE A & δ

DAMPED HARMONIC MOTION

LINEAR RESTORING FORCE $F_{RESTORE} = -kx$

VELOCITY - DEPENDENT DRAG $F_{RESIST} = -b\dot{x}$

NSL: $\sum F = m\ddot{x}$

$-kx - b\dot{x} = m\ddot{x}$

DE: $\ddot{x} + 2\beta\dot{x} + \omega_N^2 x = 0$, $\omega_N^2 = \frac{k}{m}$, $2\beta = \frac{b}{m}$

SOLUTION: $x = e^{-\beta t}\left[A_1 e^{\sqrt{\beta^2 - \omega_N^2}\, t} + A_2 e^{-\sqrt{\beta^2 - \omega_N^2}\, t}\right]$

UNDER DAMPED $\Rightarrow \omega_N^2 > \beta^2$

CRITICALLY DAMPED $\Rightarrow \omega_N^2 = \beta^2$

OVER DAMPED $\Rightarrow \omega_N^2 < \beta^2$

UNDERDAMPED MOTION ($\omega_N^2 > \beta^2$)

SOLUTION: $x(t) = Ae^{-\beta t}\cos(\omega_S t - \delta)$

WHERE $\omega_N = \sqrt{\frac{k}{m}}$, $\beta = \frac{b}{2m}$, $\omega_S = \sqrt{\omega_N^2 - \beta^2}$

BOTH A & δ FROM INITIAL CONDITIONS

ENERGY: $E(t) = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}kx^2 \neq$ CONSTANT

DECREMENT OF MOTION

$$\frac{A(t_0)}{A(t_0+t)} = \frac{Ae^{-\beta t_0}}{Ae^{-\beta(t_0+t)}} = e^{\beta t}$$

CRITICALLY DAMPED MOTION ($\omega_N^2 = \beta^2$)

SOLUTION: $x(t) = (A+Bt)e^{-\beta t}$

=> NON-OSCILLATORY

OVER DAMPED MOTION ($\omega_N^2 < \beta^2$)

SOLUTION: $x(t) = e^{-\beta t}\left[A_1 e^{\omega_{OD} t} + A_2 e^{-\omega_{OD} t}\right]$

WHERE $\omega_{OD} = \sqrt{\beta^2 - \omega_N^2}$

=> NON-OSCILLATORY

DRIVEN HARMONIC MOTION

LINEAR RESTORING FORCE: $F_{RESTORE} = -kx$

VELOCITY-DEPENDENT DRAG: $F_{RESISITNG} = -b\dot{x}$

SINUSOIDAL DRIVING FORCE: $F_{DRIVING} = F_0\cos(\omega_D t)$

WHERE ω_D IS THE DRIVING FREQUENCY

NSL: $\sum F = m\ddot{x}$

$$F_0\cos(\omega_D t) - kx - b\dot{x} = m\ddot{x}$$

DE: $\ddot{x} + 2\beta\dot{x} + \omega_N x = \left(\frac{F_0}{m}\right)\cos(\omega_D t)$

DRIVEN HARMONIC MOTION (CONTINUED)

SOLUTION: $$x(t) = x_c(t) + \frac{(F_0/m)\cos(\omega_D t - \delta)}{\sqrt{(\omega_N^2 - \omega_D^2)^2 + 4\beta^2\omega_D^2}}$$

$$\delta = \tan^{-1}\left(\frac{2\beta\omega_D}{\omega_N^2 - \omega_D^2}\right)$$

$x_c(t)$ = COMPLIMENTARY SOLUTION TO THE HOMOGENEOUS EQUATION (ON p. 2)

RESONANCE

AMPLITUDE: $\omega_D = \sqrt{\omega_N^2 - 2\beta^2} = \omega_{RESONANCE}$

MECHANICAL - ELECTRICAL EQUIVALENCE

CIRCUIT COMPONENTS MODEL MECHANICAL SYSTEMS

DISPLACEMENT	x	$\longleftrightarrow$ Q	CHARGE
VELOCITY	v	I	CURRENT
MASS	m	L	INDUCTANCE
COMPLIANCE	$1/k$	C	CAPACITANCE
DAMPING	b	R	RESISTANCE
APPLIED FORCE	F_0	$\mathcal{E}_0$	SUPPLIED emf
β:	$\frac{b}{2m}$	$\frac{R}{2L}$	
ω_N	$\sqrt{\frac{k}{m}}$	$\sqrt{\frac{L}{C}}$	

KIRCHHOFF'S LAW OF LOOPS $\Sigma V + \Sigma \mathcal{E} = 0$ GIVES

$$L\ddot{Q} + R\dot{Q} + \frac{1}{C}Q = \mathcal{E}_0 \cos\omega_D t$$

WHICH LOOKS LIKE

$$m\ddot{x} + b\dot{x} + kx = F_0 \cos\omega_D t$$

AND IS SOLVED IN EXACTLY THE SAME WAY!

COOL!