1. Briefly and in your own words (not the textbook’s) explain what a functional dependency \( \alpha \rightarrow \beta \) on a relation \( R \) means.

Whenever two tuples agree on \( \alpha \) they agree on \( \beta \).

2. If the functional dependency \( \alpha \beta \rightarrow \gamma \) is valid for a relation schema \( R \) can we conclude that the dependency \( \beta \rightarrow \gamma \) is also valid? Show why or why not.

No. A simple counterexample is easy to construct. Look at the table in question 4 and let \( \alpha = A, \beta = B, \) and \( \gamma = C \).

3. Is the following relation in First Normal Form? Briefly explain why or why not.

<table>
<thead>
<tr>
<th>PNO</th>
<th>Pname</th>
<th>Available colors</th>
<th>City</th>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Nut</td>
<td>Red, blue, green</td>
<td>Paris</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>Bolt</td>
<td>Orange</td>
<td>NYC</td>
<td>6</td>
</tr>
</tbody>
</table>

No, Available colors is non-atomic.

4. Consider the relation schema \( R = (A,B,C) \) and an instance \( r \) below:

\[
\begin{array}{ccc}
A & B & C \\
\text{a1} & \text{b1} & \text{c1} \\
\text{a2} & \text{b3} & \text{c3} \\
\text{a3} & \text{b2} & \text{c2} \\
\text{a3} & \text{b1} & \text{c4} \\
\end{array}
\]

a. What non-trivial functional dependencies could currently hold for \( r \)?

There are six, \( C \rightarrow A, C \rightarrow B, C \rightarrow AB, AB \rightarrow C, AC \rightarrow B, BC \rightarrow A \)

b. What are the possible candidate keys for \( r \)?

\( C \)

c. What are the possible super-keys for \( r \)?

\( C, AB, BC, AC, ABC \)

5. Consider the following set \( F \) of functional dependencies on a relation scheme \( R=(A,B,C) \).

\[ F = \{ A \rightarrow BC, B \rightarrow C \} \]

a. Compute the closure \( F^+ \) of \( F \). (Do not include trivial dependencies)

\[ F^+ = \{ A \rightarrow BC, B \rightarrow C, A \rightarrow B, A \rightarrow C, AC \rightarrow BC, AB \rightarrow BC, AB \rightarrow C, AC \rightarrow B, AB \rightarrow AC \} \]
b. Is $F$ a canonical cover for $R$? If it is explain why, if it is not, give the canonical cover.

No, $\{ A \rightarrow B, B \rightarrow C \}$ is the canonical cover.

6. Consider the following set of dependencies $F$ on a relation scheme $R = (V,W,X,Y,Z)$.

$F = \{ V \rightarrow WX, XY \rightarrow Z, W \rightarrow Y, Z \rightarrow V \}$

a. Compute the attribute closure $Z^+$ of $F$.

$Z^+ = \{ V, W, X, Y, Z \}$

b. Is $Z$ a candidate key? Show why or why not?

Yes, it determines every other attribute.

c. Is there another candidate key? Explain why or why not?

Yes, $V$ is as well. Compute $V^+$ to see.

7. Given the dependencies from question 6 is the decomposition $(W, X, Z)$ and $(V, Y, Z)$ lossless? Show why or why not.

Let $R_1 = \{ W, X, Z \}$ and $R_2 = \{ V, Y, Z \}$ since $R_1 \cap R_2 = \{ Z \}$ and $Z$ is a candidate key by 6a then $R_1 \cap R_2 \rightarrow R_1$ so the decomposition is lossless. By the way, $R_1 \cap R_2 \rightarrow R_2$ also, but only one is required to hold. Note: I did not ask that the decomposition be dependency preserving.

8. Is the functional dependency $WX \rightarrow V$ valid for the set of dependencies in 6? Show why or why not.

Yes, $V$ is in $\{ WX \}^+$

9. Give a lossless decomposition of $R$ from question 6 into BCNF.

There are lots of correct answers, but many that are wrong as. In particular make sure you don’t put $W$ and $Y$ in the same relation since $W \rightarrow Y$ and $W$ is not a super-key. So $R_1 = (V, X, Y, Z)$ and $R_2 = (W, Z)$ will do.

10. Consider a university database that keeps track of current students, courses, and academic departments. A student has a name and an ID. A course has a name, a number, and a department. A department has a name. Students take one or more courses, departments offer one or more courses, and departments have zero or more students as majors. A course can be either a lecture course or a lab course. Functional requirements include needing to know what courses a department offers, what students are enrolled in what courses, current majors in a department, and looking up information about specific students, courses, and departments.

a. Draw an E-R diagram that best represents the university database.

b. What are the relation schemas for your database?

Everyone got this correct.