Newton’s Third Law and Conservation of Momentum
Fall 2015

Introduction

This experiment is adapted from Realtime Physics by David Sokoloff, Ronald K. Thornton and Priscilla W. Laws. The purpose of this experiment is to study the forces between objects that interact with each other, especially in collisions, and to examine the consequences of Newton’s third law as applied to interaction forces between objects.

Investigation 1: Momentum

In this investigation we are going to develop the concept of momentum to predict the outcome of collisions. But you don’t officially know what momentum is because we haven’t defined it yet. Let’s start by predicting what will happen as a result of a simple one-dimensional collision. This should help you figure out how to define momentum and to enable you to describe collisions in mathematical terms.

Scenario: It is early fall and you are driving along a two-lane highway in a rented moving van. It is full of all of your possessions, so you and the loaded truck weigh 8000 lbs. You have just slowed down to 15 mph because you are in a school zone. It is a good thing you thought to do that, because a group of first graders are just starting to cross the road. Just as you pass the children, you see a 2000 lb sports car in the other lane heading straight for the children at about 80 mph. A desperate thought crosses your mind. You just have time to swing into the other lane and speed up a bit before making a head-on collision with the sports car. You want your truck and the sports car to crumple into a heap that sticks together and doesn’t move. Can you save the children or is this just a suicidal act?

1. Calculation: How fast would you have to be going to completely stop the sports car? What concept did you employ here to calculate your answer? You don’t need to convert units here!
Simulation: To simulate this situation you can use two carts of different masses that are set up to stick together, as shown below. You will need:

- A “light” cart (no additional mass)
- A “heavy” cart with four times the mass (put 3 masses on the cart)
- Velcro on carts aligned so that they stick together, as shown
- Level track

2. Try it: Try some head-on collisions with the carts of different mass to simulate the event on a small scale. Remove the force probes that you might find on both carts. Place the heavy cart on the left, and the light cart on the right of the track. Be sure that the carts stick together after the collision. Observe qualitatively what combinations of velocities cause the two carts to be at rest after the collision. Discuss the results of each collision, noting the direction and magnitude of the velocity of the carts after collision. What works best?

<table>
<thead>
<tr>
<th>Heavy Cart</th>
<th>Light Cart</th>
<th>What Happens?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fast</td>
<td>Slow</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Same Speed</td>
</tr>
<tr>
<td>Slow</td>
<td>Fast</td>
<td></td>
</tr>
</tbody>
</table>

3. Question 1: What happens when the lighter cart is moving much faster than the heavy cart? Much slower?

4. Question 2: Based on your prediction and your observations, what mathematical equation might you use to describe the momentum you would need to stop an oncoming vehicle traveling with a known mass and velocity? Should it depend on the mass, the velocity or both? Explain your choice.
Investigation 2: Forces Between Interacting Objects

There are many situations where objects interact with each other, for example, during collisions. In this investigation we want to compare the forces exerted by the objects on each other. In a collision, both objects might have the same mass and be moving at the same speed, or one object might be much more massive, or they might be moving at very different speeds. What factors might determine the forces the objects exert on each other? Is there some general law that relates these forces?

Activity 2-1: Collision Interaction Forces

What can we say about the forces two objects exert on each other during a collision?

1. Prediction: Suppose two objects have the same mass and are moving toward each other at the same speed so that \( m_A = m_B \) and \( \mathbf{v}_A = -\mathbf{v}_B \) (same speed, opposite direction).

Predict the relative magnitudes of the forces between object A and object B during the collision. Place a check next to your prediction.

___ Object A exerts a larger force on object B.
___ The objects exert the same size force on each other.
___ Object B exerts a larger force on object A.

2. Prediction: Suppose the masses of two objects are the same and that object A is moving toward object B, but object B is at rest. \( m_A = m_B \) and \( \mathbf{v}_A \neq 0, \mathbf{v}_B = 0 \)

Predict the relative magnitudes of the forces between object A and object B during the collision.

___ Object A exerts a larger force on object B.
___ The objects exert the same size force on each other.
___ Object B exerts a larger force on object A.
3. **Prediction**: Suppose the mass of object A is greater than that of object B and that it is moving toward object B, which is at rest.

\[ m_A > m_B \quad \text{and} \quad \vec{v}_A \neq 0, \vec{v}_B = 0 \]

Predict the relative magnitudes of the forces between object A and object B during the collision.

___ Object A exerts a larger force on object B.
___ The objects exert the same size force on each other.
___ Object B exerts a larger force on object A.

Provide a summary of your predictions. What are the circumstances under which you predict that one object will exert a greater force on the other object?

**Simulation**: To test the predictions you made you can study gentle collisions between two force probes attached to carts. You can add masses to one of the carts so it has significantly more mass than the other. You will need the following equipment:

- Logger Pro software
- Two low-friction carts
- Two force probes with rubber tips
- 1 mass block to double the cart mass
- Level track

4. Set up the apparatus as shown in the following diagram; be sure that the force probe on cart A is connected to CH1 of the computer interface, and force probe B is connected to CH2. No mass block is needed yet:

Check that the force probes are securely fastened to the carts. The rubber tips should be carefully aligned so that they will collide head-on with each other. Be sure that the switch on the force probes is set to ±50N.

5. Start Logger Pro and open the experiment file called **Collisions SLU** from T:\Phys103. The software is set up to measure the forces applied to each probe with a very fast data collection rate of 4000 points per second. (This allows you to see all of the details of the collision which takes place in a very short time interval.)
6. Now you will reverse the sign of force probe A (the cart on your left), since a push on it is negative (toward the left), as follows: Click the Experiment menu, then Set Up Sensors, then Lab Pro:1. Click the photo of the force probe connected to CH1, and choose Reverse Direction. Click the × to close the window.

7. Use the two carts to explore various situations that correspond to the predictions you made about interaction forces. Your goal is to find out under what circumstances one cart exerts more force on the other. Try collisions (a) – (c) listed below. Some additional info:

* Be sure to zero the force probes before each collision! Click the Zero button (next to the green Collect button) and check the box next to both force probes. If your collision exceeds 20 N, try it again, but more gently.
* Click the Collect button to start. When the button turns red, collide the carts. You do not need to hurry; data collection will not start until the carts actually collide.
* Sketch the graphs for each collision on the appropriate axes on pages 6 and 7. Be sure to label your graphs.
* For each collision you will use Logger Pro to find the values of the maximum force and impulse exerted by each cart on the other (recall from last week that impulse = FΔt, the area under a force-time graph):
  - Click the top graph to select it. Click and drag over the region to be measured.
  - Click the Analyze menu, then Integral. A box appears with the measured impulse.
  - Click the bottom graph (the same region is already highlighted) and again choose Analyze, Integral.
  - Again click the Analyze menu, and choose Statistics. Record the maximum force for each.
* Record these force and impulse values in the space to the right of each graph, as well as the percent difference between the forces.

8. In the box that appears below the graphs on pages 6 and 7, carefully describe what you observed about the direction and velocity of the carts after collision.
Collisions:

a. Two carts of the same mass moving toward each other at about the same speed.

Collision Observations (direction & velocity):

b. Two carts of the same mass, one at rest and the other moving toward it.

Collision Observations (direction & velocity):
c. Put one mass block on cart A; this will make cart A twice as massive as cart B. Push cart A toward cart B, which is initially at rest.

Impulse: __________
Max. Force: __________

Impulse: __________
Max. Force: __________

% Diff (F): __________

Collision Observations (direction & velocity):

8. Question 1: Did your observations agree with your predictions? What can you conclude about forces of interaction during collisions? How do forces compare on a moment by moment basis during each collision?

9. Question 2: You have studied Newton’s third law in lecture. Do your conclusions have anything to do with Newton’s third law? Explain.

10. Question 3: How does the impulse due to cart A acting on cart B compare to the impulse of cart B acting on cart A in each collision? Are they the same in magnitude or different? Do they have the same sign or different signs? (Recall that you reversed one of the force probes!)
Investigation 3: Newton's Laws and Momentum Conservation

Your previous work should have shown that interaction forces between two objects are equal in magnitude and opposite in sign (direction) on a moment by moment basis for all the interactions you might have studied. This is a testimonial to the seemingly universal applicability of Newton’s third law to interactions between objects.

As a consequence of the forces being equal and opposite at each moment, you should have seen that the impulses of the two forces were always equal in magnitude and opposite in direction. This observation, along with the impulse-momentum theorem, is the basis for the derivation of the conservation of momentum law. (The impulse-momentum theorem is really equivalent to Newton’s second law since it can be derived mathematically from the second law.) The argument is that Impulse$_{A}$ acting on cart A during the collision equals the change in momentum of cart A, and Impulse$_{B}$ acting on cart B during the collision equals the change in momentum of cart B:

\[
\text{Impulse}_A = \Delta \vec{p}_A \quad \text{and} \quad \text{Impulse}_B = \Delta \vec{p}_B
\]

But, as you have seen, if the only forces acting on the carts are the interaction forces between them and the time of the interaction is the same, then

\[
\text{Impulse}_A = F_{AB} \Delta t = -F_{BA} \Delta t = \text{Impulse}_B \quad \Delta \vec{p}_A = -\Delta \vec{p}_B
\]

or

\[
\Delta \vec{p}_A + \Delta \vec{p}_B = 0
\]

i.e., there is no change in the total momentum $\Delta \vec{p}_A + \Delta \vec{p}_B$ of the system (the two carts).

If the momenta of the two carts before (initial – subscript $i$) and after (final – subscript $f$) the collision are represented in the diagrams below, then $\vec{p}_f = \vec{p}_i$.

\[
\vec{p}_A = m_A \vec{v}_{Ai} \quad \vec{p}_B = m_B \vec{v}_{Bi}
\]

\[
\vec{p}_A = m_A \vec{v}_{Af} \quad \vec{p}_B = m_B \vec{v}_{Bf}
\]

where $\vec{p}_i = m_A \vec{v}_{Ai} + m_B \vec{v}_{Bi} \quad \text{and} \quad \vec{p}_f = m_A \vec{v}_{Af} + m_B \vec{v}_{Bf}$
Simulation: In the next activity you will examine whether momentum is conserved in a simple inelastic collision between two carts of unequal mass. You will need the following:

- Logger Pro software
- Two force probes with rubber tips
- Motion detector and index card
- Two low-friction carts with Velcro
- 1 mass block to place on one cart to double its mass
- Level ramp

Activity 3-1: Conservation of Momentum in an Inelastic Collision

1. Set up the carts, ramp, and motion detector as shown below, taping an index card to the back of cart A; this will provide a flat surface for the motion detector to read. Arrange the carts so that their Velcro sides will stick together after the collision. Remove the force probes from the carts, but leave the mounting post in place. Add one mass block to cart A; use a bolt and wing nut to attach the mass block to the left-most side of cart A as shown above (this will help the motion detector produce a good signal).

2. Measure the mass of cart A (with its extra mass block attached) and cart B:

   \[ m_A = \] \[ m_B = \]

3. Prediction 1: You are going to push the more massive cart A and collide it with cart B, which is initially at rest. The carts will stick together after the collision. Suppose that you measure the total momentum of cart A and cart B before and after the collision. How do you think that the total momentum after the collision will compare to the total momentum before the collision? Explain the basis for your prediction.

4. Try it. Open the experiment file called Inelastic Collision SLU from the T: drive. Remove cart B from the track, and start cart A with its left side around the 15 cm mark of the track. Click the Collect button, and push the cart when you hear the motion detector begin to click. This way you can see what happens with the motion of just one cart (make sure you understand what this graph is indicating).

5. Now you will collide the carts together. Place cart B on the track, with the left side of the cart at the 80 cm mark. The left side of cart A should again be at the 15 cm mark. Click the Collect button, and when you hear the clicks of the motion detector, give cart A a brisk push toward cart B and release it. Be sure that the motion detector does not see your hand. Repeat until you get a good run when the carts stick and move together after the collision (check with your instructor).
6. Sketch the graph on the axes below, and draw brackets on the graph to label the region where:
   a. cart A moves at a constant velocity before collision
   b. the collision between the carts occurs
   c. both carts move together at a constant velocity after collision

   ![Graph with axes and brackets indicating regions](image)

a. Use Logger Pro to measure the velocity of cart A over a short interval of time just before the collision and the velocity of the two carts together over a short interval of time just after the collision:

   a. Click and drag over a small region of the graph where cart A moves before collision, and then choose Analyze, and then Statistics. Record the average (mean) velocity below.
   b. Repeat for the region after collision occurs. Be sure to find the average velocities over time intervals just before and just after – but not during – the collision.

   \[ \vec{v}_{Ai} = \quad \vec{v}_{Af} = \vec{v}_{Bf} = \]

8. Calculate the total momentum of carts A and B before and after the collision, as well as the percent difference. Show your calculations below.

   \[ \vec{p}_i = \quad \vec{p}_f = \quad \% \text{diff} = \]

9. Arrange the statistics boxes on the graph so they don’t overlap, and print your graph in landscape mode; be sure it includes the measurements of your velocities.

10. Question 1: Was momentum conserved during the collision? Was there a slight loss or gain in momentum? Did your results agree with your prediction? Explain.

11. Question 2: What are the difficulties in doing this experiment that might cause errors in the results?