25 points

Newton's Third Law and Conservation of Momentum[1]

Fall 20xx

Name: Solution

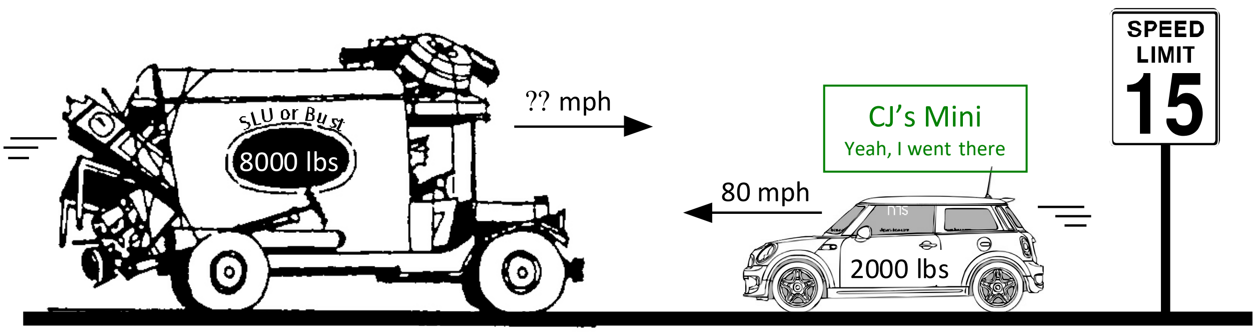
Partners:

Introduction

The purpose of this experiment is to study the forces between objects that interact with each other, especially in collisions, and to examine the consequences of *Newton's third law* as applied to interaction forces between objects. *Write all of your answers in the space provided within these instructions. Write in complete sentences, and be sure to show your work. You won't be penalized for incorrect predications; however, you will lose points if your results don't match your prediction, and you say that it does!*

Investigation 1: Momentum

In this investigation we are going to develop the concept of momentum to predict the outcome of collisions. But you don't officially know what momentum is because we haven't defined it yet. Let's start by predicting what will happen as a result of a simple one-dimensional collision. This should help you figure out how to define momentum and to enable you to describe collisions in mathematical terms.



Scenario: It is early fall and you are driving along a two-lane highway in a rented moving van. It is full of all of your possessions, so you and the loaded truck weigh 8000 *lbs*. You have just slowed down to 15 *mph* because you are in a school zone. It is a good thing you thought to do that, because a group of first graders are just starting to cross the road. Just as you pass the children, you see a 2000 *lb* sports car in the other lane heading straight for the children at about 80 *mph*. A desperate thought crosses your mind. You just have time to swing into the other lane and speed up a bit before making a head-on collision with the sports car. You want your truck and the sports car to crumple into a heap that sticks together and doesn't move. Can you save the children or is this just a suicidal act?

1. *Calculation*: How fast would you need to be going to completely stop the sports car? What concept did you employ here to calculate your answer? *You don't need to convert units here!* Conservation of momentum – both cars need to have the same momentum!

$$p_{van} + p_{car} = 0$$
$$p_{van} = -p_{car}$$
$$m_v v_v = -m_c v_c$$
$$v_v = \frac{-m_c v_c}{m_v}$$

So:

$$p_{car} = m_c v_c = 2000(-80) = -160{,}000\ {}^{lb\cdot mi}/_{hr}$$
$$v_{van} = \frac{-p_{car}}{m_v} = \frac{-(-160{,}000)}{8000} = +20\ {}^{mi}/_{hr}$$

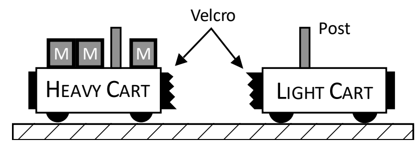
The van must speed up!

Wrong: -1
Drop negative on velocity: -0.3
Just wrote $m_1v_1 = m_2v_2$: -0.2
Wrote $m_1v_1 = m_2v_2$ and dropped sign for car velocity: -0.5
No answer for 'concept': -0.4
Correct answer but "won't save children": -0.2

[1] This experiment is adapted from **Realtime Physics** by David Sokoloff, Ronald K. Thornton and Priscilla W. Laws.

Simulation: To simulate this situation you can use two carts of different masses that are set up to stick together, as shown below. You will need:

- A "light" cart (no additional mass)
- A "heavy" cart with four times the mass (put 3 masses on the cart)
- Velcro on carts aligned so that they stick together
    - Level track



2. *Try it*: Try some head-on collisions with the carts of different mass to simulate the event on a small scale. Remove the force sensors from both carts. Place the heavy cart on the left, and the light cart on the right of the track. Be sure that the carts stick together after the collision. Observe *qualitatively* what combinations of velocities cause the two carts to be at rest (both carts have stopped) after the collision. Discuss the results of each collision, noting the direction *and* magnitude of the velocity of the carts after collision. (*Keep trying until both carts stop on impact*.)

Heavy Cart	Light Cart	What Happens?
Fast	Slow	Lighter cart moves backward (to the right) quickly.
Same Speed		Lighter cart moves backward (to the right) more slowly than before.
Slow	Fast	Both carts come to a dead stop!

No mention of velocity: -0.5
Carts didn't stop: -1
Skipped one velocity: -0.25

3. *Question 1*: Which combination was best to bring both carts to rest after collision?

Moving the heavy cart slowly and the light cart quickly will stop both carts on impact

4. *Question 2*: Based on your prediction and your observations, what mathematical equation might you use to describe the momentum you would need to stop an oncoming vehicle traveling with a known mass and velocity? Should it depend on the mass, the velocity or both? Explain your choice.

Skipped this first equation: -0.1

Sign: -0.2
Not quite complete: -0.3
Just wrote p = mv: -0.5
Didn't write that $v_f = 0$: -0.2

$p_{1i} + p_{2i} = p_{1f} + p_{2f}$

$p_{1i} + p_{2i} = 0$ {since velocity after collision equals zero}

$p_{1i} = - p_{2i}$ Depends on ***both*** mass and velocity

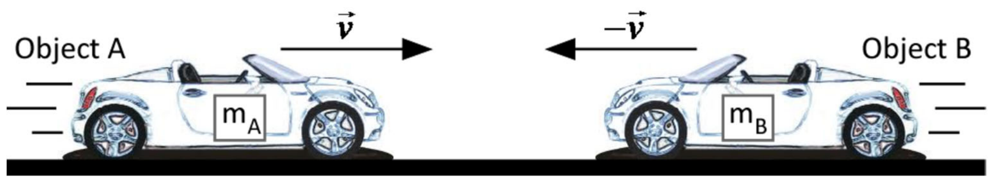
Investigation 2: Forces Between Interacting Objects

There are many situations where objects interact with each other, for example, during collisions. In this investigation we want to compare the forces exerted by the objects on each other. In a collision, both objects might have the same mass and be moving at the same speed, or one object might be much more massive, or they might be moving at very different speeds. What factors might determine the forces the objects exert on each other? Is there some general law that relates these forces?

Activity 2-1: Collision Interaction Forces

What can we say about the forces two objects exert on each other during a collision?

1. *Prediction:* Suppose two objects have the same mass and are moving toward each other at the same speed so that $m_A = m_B$ and $\vec{v}_A = \vec{v}_B$ (same speed, opposite direction).



Predict the relative magnitudes of the forces between object A and object B during the collision. Place a check next to your prediction.

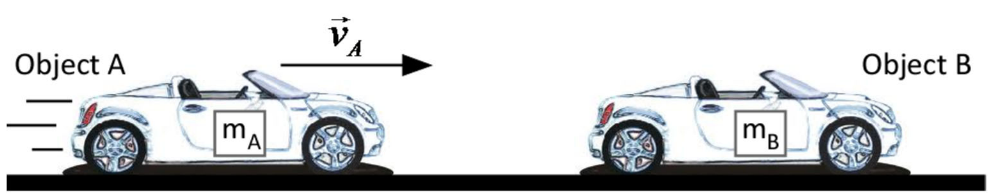
___ Object A exerts a larger force on object B.

_×_ The objects exert the same size force on each other.

___ Object B exerts a larger force on object A.

2. *Prediction:* Suppose the masses of two objects are the same and that object A is moving toward object B, but object B is at rest.

$$m_A = m_B \quad and \quad \vec{v}_A \neq 0, \vec{v}_B = 0$$



Predict the relative magnitudes of the forces between object A and object B during the collision.

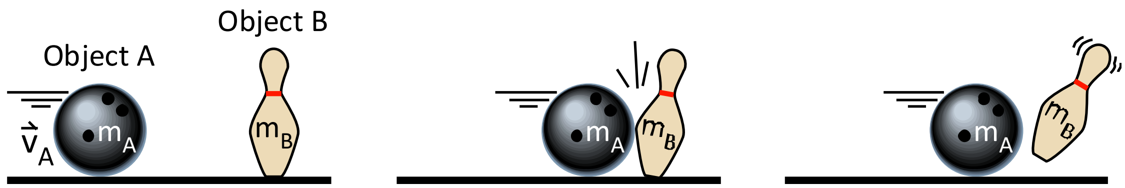
___ Object A exerts a larger force on object B.

_×_ The objects exert the same size force on each other.

___ Object B exerts a larger force on object A.

3. *Prediction*: Suppose the mass of object A is greater than that of object B and that it is moving toward object B, which is at rest.

$$m_A > m_B \quad and \quad \vec{v}_A \neq 0, \vec{v}_B = 0$$



Predict the relative magnitudes of the forces between object A and object B during the collision.

___Object A exerts a larger force on object B.

_×_The objects exert the same size force on each other.

___Object B exerts a larger force on object A.

Provide a summary of your predictions. What are the circumstances under which one object will exert a greater force on the other object?

Note: Number of my students who initially got all three predictions correct:

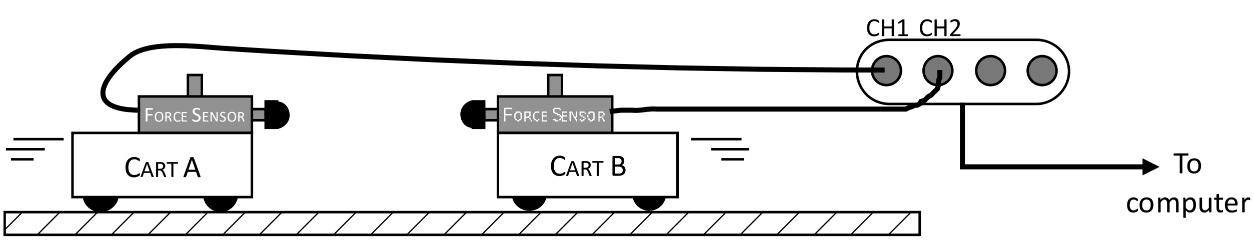
(2016) Tuesday: 2/6		Wednesday: 0/16
(2017) Monday: 12/14		Wednesday: 6/12
(2018) Monday: 4/14		Wednesday: 3/10
(2019) Monday: 1/10		Wednesday: 4/7
(2021) Tuesday: 4/10		Wednesday: 0/14
(2022) Mon: 6/11	Tue: 0/6	Wednesday: 5/9

Forces are always equal during collisions

Simulation: To test the predictions you made you can study *gentle* collisions between two force sensors attached to carts. You can add masses to one of the carts so it has significantly more mass than the other. You will need the following equipment:

- Logger Pro software
- Two low-friction carts
- Two force sensors with rubber tips
- 1 mass block to double the cart mass
- Level track

4. Set up the apparatus as shown in the following diagram: be sure that the cart A force sensor connects to CH1 of the computer interface, and force sensor B connects to CH2. Remove all mass blocks:



Securely fastened the force sensors on the carts. The rubber tips should be *carefully aligned* so that they will collide head-on with each other. Be sure that the switch on the force sensors is set to ±50N.

5. Start Logger Pro and open the experiment file called **Collisions SLU** from T:\Phys103. The software is set up to measure the forces applied to each sensor with a very fast data collection rate of 4000 points per second. (This allows you to see all of the details of the collision which takes place in a very short time interval.)

6. Now you will reverse the sign of force sensor A (the cart on your left), since a push on it is negative (toward the left), as follows: Click the *Experiment* menu, then *Set Up Sensors*, then *Lab Pro:1*. Click the photo of the force sensor connected to CH1, and choose *Reverse Direction*. Click the × to close the window.
7. Use the two carts to explore various situations that correspond to the predictions you made about interaction forces. Your goal is to find out under what circumstances one cart exerts more force on the other. Try collisions (a) – (c) listed on the next page. Some additional info:

- **Be sure to zero the force sensors <u>before each collision</u>!** Click the **Zero** button (next to the green Collect button) and check the box next to both force sensors. Try to get between 10 *N* and 20 *N* during impact; if your collision exceeds 20 *N*, try it again, but more gently.
- Click the **Collect** button to start. When the button turns red, collide the carts. *You do not need to hurry; data collection will not start until the carts actually collide.*

Didn't print graph: -0.5

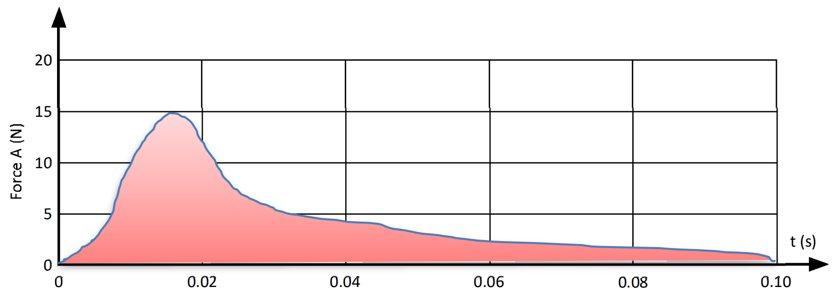
- Print a copy of just the first collision graph for you and your lab partner. Sketch all three collision graphs on the appropriate axes on the next two pages.
- For each collision you will use Logger Pro to find the values of the maximum force and *impulse* exerted by each cart on the other (recall from last week that impulse = $F\Delta t$, the area under a force-time graph):
    - Click the top graph to select it. Click and drag over the region to be measured (the time span over which the collision occurs.)
    - Click the *Analyze* menu, then *Integral*. A box appears with the measured impulse.
    - Click the bottom graph (the same region is already highlighted) and again choose *Analyze*, *Integral*.
    - Again click the *Analyze* menu, and choose *Statistics*. Record the maximum force for each.
- Record these force and impulse values in the space to the right of each graph, as well as the percent difference between the forces.

8. In the box that appears below the graphs on pages 6 and 7, <u>carefully</u> describe what you observed about the direction and velocity of the carts after collision.

- *Note that the force sensors are not identical. There might be a difference of up to 5% due to a difference in the calibration of the force sensors used.*

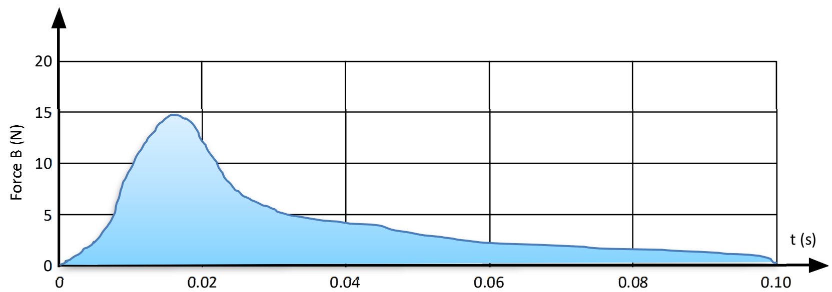
Collisions:

a. Two carts of the same mass moving toward each other at about the same speed.



Impulse: 0.2216 s•N

Max. Force: 15.5 N



Impulse: 0.2205 s•N

Max. Force: 15.5 N

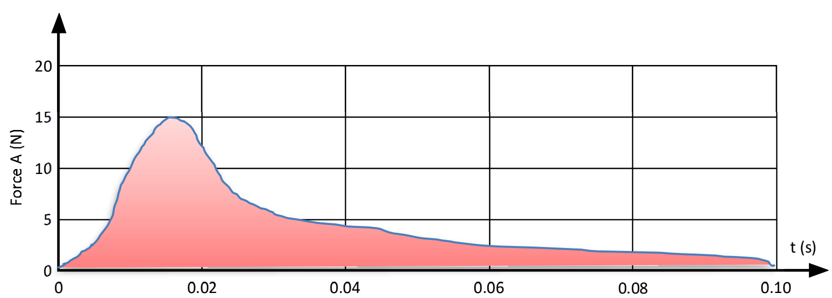
% Diff (F): 0

Collision Observations (direction & velocity):

+1 point (0.5 each, dir & vel)

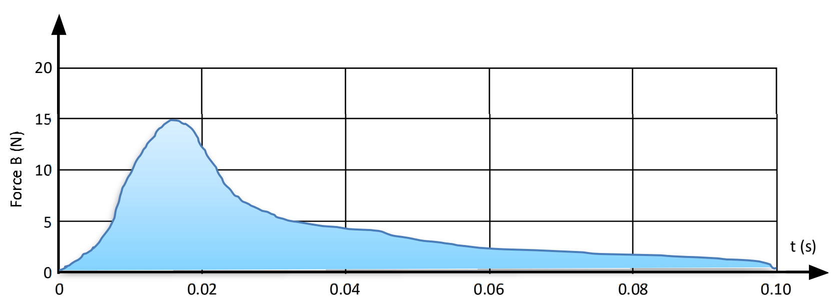
Blah, blah, blah

b. Two carts of the same mass, one at rest and the other moving toward it.



Impulse: 0.2067 s•N

Max. Force: 12.5 N



Impulse: 0.2052 s•N

Max. Force: 12.3 N

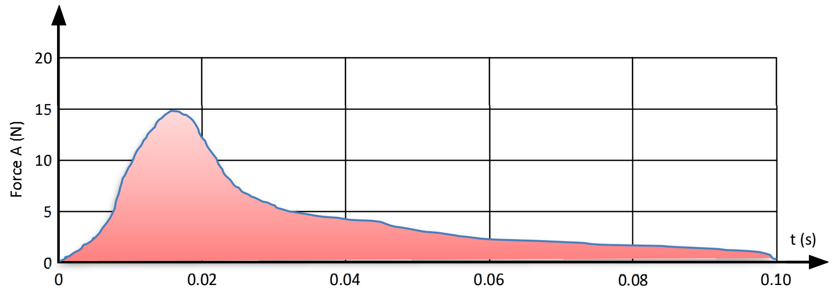
% Diff (F): 2%

Collision Observations (direction & velocity):

+1 point (0.5 each, dir & vel)

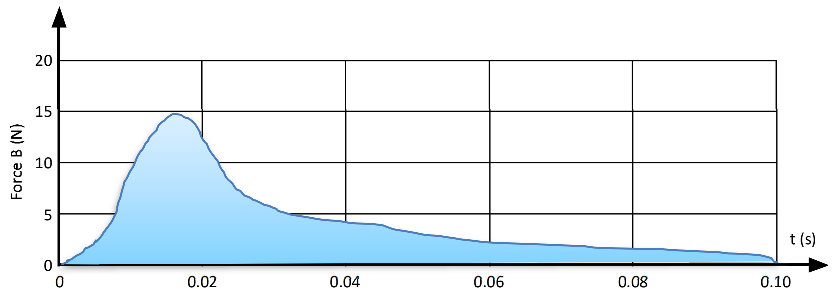
Blah, blah, blah

c. Put one mass block on cart A; this will make cart A twice as massive as cart B. Push cart A toward cart B, which is initially at rest.



Impulse: 0.2624 s•N

Max. Force: 17.64 N



Impulse: 0.2659 s•N

Max. Force: 17.56 N

% Diff (F): 0.5%

Collision Observations (direction & velocity):

+1 point (0.5 each, dir & vel)

Blah, blah, blah

8. *Question 1*: Did your three observations agree with your three predictions? What can you conclude about forces of interaction during collisions? How do forces compare on a moment-by-moment basis during each collision?

Yes! Forces are always equal

Said 'Yes' but predications were *wrong*: -2
Said 'No but predications were *correct*: -1

9. *Question 2*: You have studied *Newton's third law* in lecture. Do your conclusions have anything to do with *Newton's third law?* Explain.

Yes! Action-reaction. Forces exerted are equal and opposite

10. *Question 3*: How does the impulse due to cart A acting on cart B compare to the impulse of cart B acting on cart A in each collision? Are they the same in magnitude or different? Do they have the same sign or different signs? (Recall that you reversed one of the force sensors!)

Same magnitude, opposite signs

Same signs: -0.5

Investigation 3: Newton's Laws and Momentum Conservation

Your previous work should have shown that interaction forces between two objects are equal in magnitude and opposite in sign (direction) on a moment by moment basis for all the interactions you might have studied. This is a testimonial to the seemingly universal applicability of *Newton's third law* to interactions between objects.

As a consequence of the forces being equal and opposite at each moment, you should have seen that the impulses of the two forces were always equal in magnitude and opposite in direction. This observation, along with the *impulse-momentum* theorem, is the basis for the derivation of the *conservation of momentum law*. (The *impulse-momentum* theorem is really equivalent to *Newton's second law* since it can be derived mathematically from the *second law.)* The argument is that $Impulse_A$ acting on cart A during the collision equals the change in momentum of cart A, and $Impulse_B$ acting on cart B during the collision equals the change in momentum of cart B:

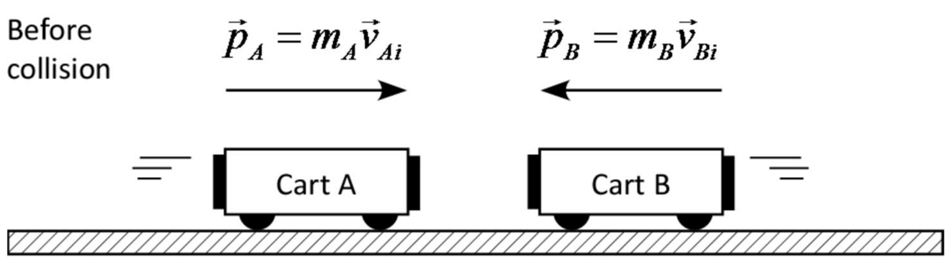
$$\text{Impulse}_A = \Delta\vec{p}_A \quad \textit{and} \quad \text{Impulse}_B = \Delta\vec{p}_B$$

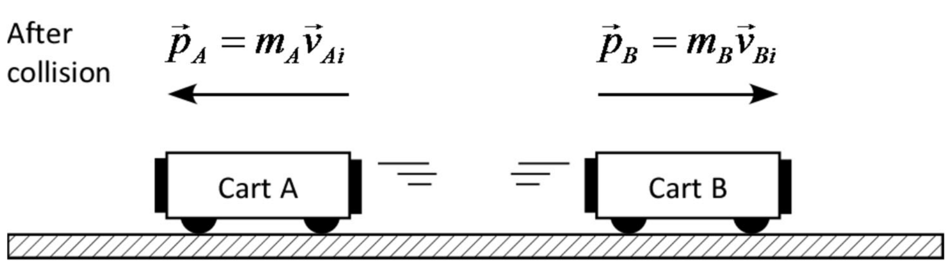
But, as you have seen, if the only forces acting on the carts are the interaction forces between them and the time of the interaction is the same, then

$$\text{Impulse}_A = F_{AB}\Delta t = -F_{BA}\Delta t = \text{Impulse}_B$$
$$\Delta\vec{p}_A = -\Delta\vec{p}_B$$
$$\textit{or}$$
$$\Delta\vec{p}_A + \Delta\vec{p}_B = 0$$

i.e., there is no change in the total momentum $\Delta\vec{p}_A + \Delta\vec{p}_B$ of the system (the two carts).

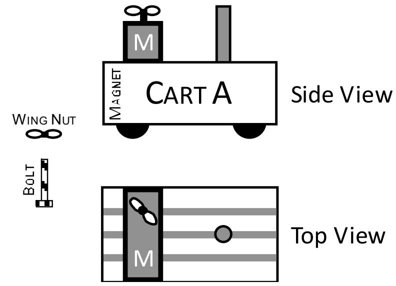
If the momenta of the two carts before (initial – subscript *i*) and after (final – subscript *f*) the collision are represented in the diagrams below, then $\vec{p}_f = \vec{p}_i$





where $\vec{p}_i = m_A\vec{v}_{Ai} + m_B\vec{v}_{Bi}$ *and* $\vec{p}_f = m_A\vec{v}_{Af} + m_B\vec{v}_{Bf}$

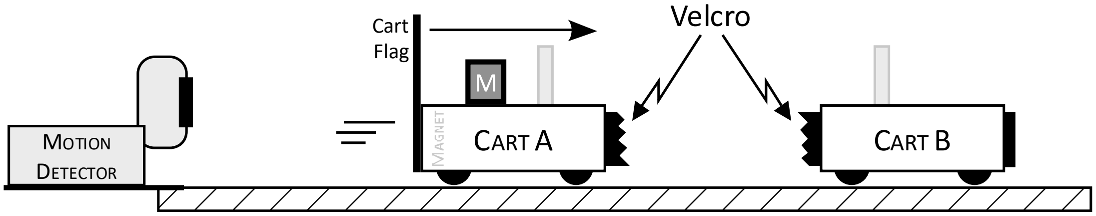
Simulation: In the next activity you will examine whether momentum is conserved in a simple *inelastic* collision between two carts of unequal mass. You will need the following:



- Logger Pro software
- Motion detector and cart flag
- Two low-friction carts with Velcro
- 1 mass block to place on one cart to double its mass
- Level track

Activity 3-1: Conservation of Momentum in an Inelastic Collision

1. Remove the force sensor from each cart, but leave the mounting post in place. Attach the cart flag to the back of cart A (on the side labeled MAGNET), arranging the carts so that their Velcro sides will stick together after the collision. Place the head of the bolt in one of the side slots of cart A, slide it into position and then add one mass block. The mass should be in a position that does not interfere with the mounting post. Tighten a wing nut on the bolt to hold the mass in place on cart A, as shown above.



2. Measure the mass of cart A (with its extra mass block attached) and cart B:

   m_A = 1.0429 kg m_B = 0.5123 kg

   No units: -0.2

3. *Prediction 1*: You are going to push the more massive cart A and collide it with cart B, which is initially at rest. The carts will stick together after the collision. Suppose that you measure the *total* momentum of cart A and cart B before and after the collision. How do you think that the total momentum after the collision will compare to the total momentum before the collision? Explain the basis for your prediction.

   Total momentum before = total momentum after

   p is conserved

4. *Try it*. Open the experiment file called **Inelastic Collision SLU** from the T: drive. Remove cart B from the track, and start cart A with its left side around the 15 *cm* mark of the track. Click the **Collect** button, and push cart A to the right when you hear the motion detector begin to click. This way you can see what happens with the motion of just one cart (make sure you understand what this graph is indicating).

5. Now you will collide the carts together. Place cart B on the track, with the left side of the cart at the 80 *cm* mark. The left side of cart A should again be at the 15 *cm* mark. Click the **Collect** button, and when you hear the clicks of the motion detector, briskly push cart A toward cart B and release it. *Be sure that the motion detector does not see your hand.* Repeat until you get a good run when the carts stick and move together after the collision (*check with your instructor*).

6. Use Logger Pro to measure the velocity of cart A over a short interval of time (~0.1 *sec*) just *before* the collision and the velocity of the two carts together over a short interval of time just *after* the collision:

   a. Click and drag over a small (0.1 *sec*) region of the graph where cart A moves *just before* collision, and then choose *Analyze*, and then *Statistics.* Record the average (mean) velocity below.
   b. Repeat for the region *just after* collision occurs. Be sure to find the average velocities over time intervals just before and just after – but not *during* – the collision.

Each missing unit: -0.25

$\vec{v}_{Ai} =$ 0.4184 m/s $\qquad \vec{v}_{Af} = \vec{v}_{Bf} =$ 0.2714 m/s

7. Calculate the total momentum of carts A and B before and after the collision, as well as the percent difference. Show your calculations below.

$$p_i = (m_A \cdot v_{Ai}) + (m_B \cdot v_{Bi}) \qquad \{v_{Bi} = 0\}$$
$$= 0.486 \ {}^{kg\cdot m}/_{s}$$

$$p_f = (m_A \cdot v_{Af}) + (m_B \cdot v_{Bf})$$
$$= 0.422 \ {}^{kg\cdot m}/_{s}$$

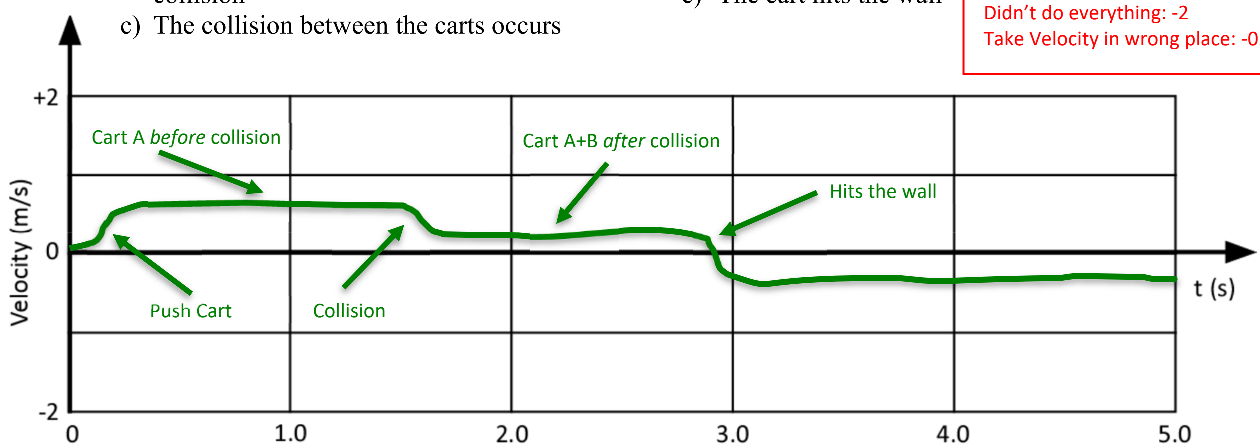
Each missing unit: -0.25

$\vec{p}_i =$ 0.486 kg•m/s (+1 pt.) $\qquad \vec{p}_f =$ 0.422 kg•m/s (+1 pt.) $\qquad \% \ diff =$ 3% loss (+0.5 pt.)

8. Arrange the statistics boxes on the graph so they don't overlap, and print your graph in landscape mode; be sure it includes the measurements of your velocities. Print a graph for each lab partner.
9. Write the following labels on your printed velocity graph: Note that students now just label their printout!

   a) Cart A is pushed
   b) Cart A moves at a constant velocity, before collision
   c) The collision between the carts occurs
   d) Both carts move together at a constant velocity, after collision
   e) The cart hits the wall

No labels: -1
Didn't do everything: -2
Take Velocity in wrong place: -0.5



10. *Question 1*: Was momentum conserved during the collision? Was there a slight loss or gain in momentum? Did your results agree with your prediction?

    Yes, within 3%

Missing *any* of the three parts of this question: -0.3 each part
"*Not conserved*" with a small loss of momentum: -0.5

11. *Question 2*: What are the difficulties in doing this experiment that might cause errors in the results?

    Some possibilities:
    a) Friction (if momentum decreases)
    b) Track not level, so cart accelerates
    c) Cart B is moving (not stationary)

Missing *friction*: -0.2