Torque & Equilibrium

Fall 20xx

Solutions

Introduction

In this experiment you will calculate the torque necessary to keep an *equal arm balance* in equilibrium. You will also balance a rod with a mass on one end and explain its motion by considering the moment of inertia of the system.

Theory

We have used Newton's Laws to talk about *equilibrium*; equilibrium means that an object is not accelerating because the sum of all the forces acting on the object is zero. In this experiment we introduce the idea of *rotational equilibrium* where an object is not rotating because the sum of the *torques* is zero. Torque can be thought of as a rotational analog of force. The Greek letter τ (tau) is used to represent torque:

$$\tau = Fr\sin\theta \qquad \textit{(Eqn. 1)}$$

where F is the applied force; the lever arm, r, is the distance from the pivot (the axis of rotation) to the point where the force is applied; and θ is the angle between r and F (Figure 1).

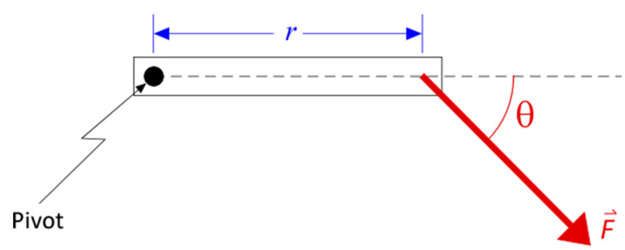

Figure 1: A lever arm

Notice that a longer lever arm results in a larger torque. A larger force also results in a larger torque, as well as a value of θ closer to 90°. A torque can cause an object to rotate in a clockwise or counterclockwise fashion. A clockwise rotation is denoted as a negative torque; a counter-clockwise rotation is denoted by a positive torque. *For an object to be in equilibrium, the sum of the torques acting on an object must equal zero*.

Experiment

Part I: Characterizing the Equal Arm Balance

The equal arm balance has seven loops from which mass can be suspended: three to the left of center, three to the right of center and one in the middle. Since we need the length of the lever arm, r, and the angle, θ for our calculations, we will begin by finding those values for our lever arm, as shown below:

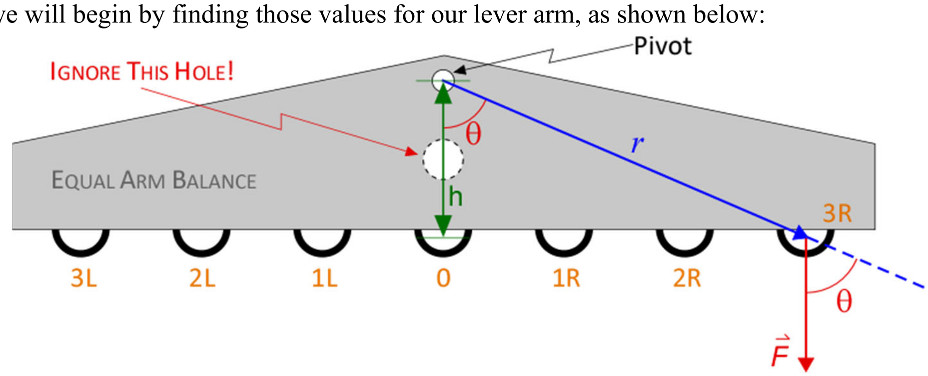

Figure 2: The Equal Arm Balance

1. Create the following table in your report; note that the length measurements are in units of meters. You don't need a measurement for hanger position 0:

Hanger position	h (m)	r (m)	$\theta = \cos^{-1}(h/r)$
1L 1R	0.051	0.077	48.5
2L 2R		0.125	65.9
3L 3R		0.178	73.4

2. Remove the equal arm balance from the clamp and use a ruler to measure h and the values of r for each hanger position 1, 2, and 3 on the left **AND** right side of the pivot. When your measurements are finished, calculate the angle, θ at which the force is applied to 1/10th of one degree. Note that h and r are measured from the center of the pivot to the center of the loop (Figure 2). Reattach the equal arm balance to the clamp when finished.

> Note that there is a hole in the equal arm balance between the pivot point and hanger position 0 (SHOWN IN FIGURE 2!) Measure h from hanger position 0 to the pivot point, **not** the hole!

Part II: Torque and Equilibrium

First you will get a qualitative feel for the amount of torque necessary to balance the arm. Then you will experimentally determine the amount of torque needed to put the lever arm in a state of equilibrium.

3. *Prediction #1*: If you suspend mass on loop 3 on one side of the equal arm balance, from which loop on the *other side* will it be easiest for you to pull straight down to balance the arm: 1, 2 or 3? Explain your choice.

Easiest to pull from loop 3 on opposite side: long r and greater θ means less force is required for same torque

4. Suspend a mass of 0.250 *kg* on the left side of the equal arm balance at position 3L (remember that the hangar has a mass of 50 *g*!) Now use your hand to balance the arm by pulling *straight down* on the right side, first at position 3R, then position 2R and finally position 1R. You should also try pulling straight down on the loop in the center (position 0).

5. Was your prediction correct? Which loop position required that you pull the hardest to balance the arm? Which required the least effort? Briefly explain the difference, in terms of the length of the lever arm, the angle θ and the force required. Yep!

Calculation of Torque

6. *Prediction #2*: If you again suspend mass on loop 3 on the left side of the equal arm balance, which loop on the right side will produce the greatest torque when you balance the apparatus? Or will the torque be the same on all three loops on the right side? Explain your answer.

   In equilibrium, the torque will be the same on both sides of the equal arm balance

7. Suspend 90 *g* from loop **3L** on the *left side* of the equal arm balance (remember that the mass hanger is 50 *g*).

8. Calculate the amount of torque that this mass exerts on the left side of the equal arm balance, using *Eqn. 1* and your measurements of the lever arm from Part I: $\tau_{3L} = (mg)r_3 \cdot \sin\theta$. *Calculate the torque to 3 significant figures.* 3L (90 g): $\tau_{3Left} = (m_{Left}\,g)\,r_{Left3} \cdot \sin\theta_3 = (0.090\ kg)\ g\ (0.178\ m)\ \sin(73.4°) = 0.150\ N{\cdot}m$

9. Now you will determine how much mass is needed on the right side to balance the apparatus. Hang a sufficient amount of mass on loop 1R (right side) so that the arm is level, again remembering the hanger mass of 50 g.

10. Calculate τ_{1R}, the amount of torque exerted on loop 1R by this mass, and then calculate the % difference between this (right side) torque and the left side torque calculated in Step 8. *You should check your measurements if the difference is much more than 1 or 2%!*

11. Again balance the arm by first hanging mass on loop **2R** and then again on loop **3R**. Each time, calculate the right side torques, and calculate the % difference of each with the torque on the left side from Step 8.

12. Repeat steps 7 – 11 with 90 *g* suspended from loop **2L**.

13. Were you correct with your answer for prediction #2? Briefly discuss your results.

3L (90 g): $\tau_{3Left} = (m_{Left}\, g)\, r_{Left3} \cdot \sin\theta_3$ = (0.090 *kg*) g (0.178 *m*) sin(73.4°) = 0.150 *N·m*

1R (270 *g*): $\tau_{Right1} = m_{Right1} g r_{Right1} \sin\theta_1$ = (0.270 *kg*) g (0.076 *m*) sin(48.5°) = 0.151 *N·m* **0.1%**

2R (135 *g*): $\tau_{Right2} = m_{Right2} g r_{Right2} \sin\theta_2$ = (0.135 *kg*) g (0.125 *m*) sin(65.9°) = 0.151 *N·m* **0.3%**

3R (90 *g*): $\tau_{Right3} = m_{Right3} g r_{Right3} \sin\theta_3$ = (0.090 *kg*) g (0.178 *m*) sin(73.4°) = 0.150 *N·m* **0.0%**

2L (90 g): $\tau_{2Left} = (m_{Left}\, g)\, r_{Left2} \cdot \sin\theta_2$ = (0.090 *kg*) g (0.125 *m*) sin(65.9°) = 0.101 *N·m*

1R (180 *g*): $\tau_{Right1} = m_{Right1} g r_{Right1} \sin\theta_1$ = (0.180 *kg*) g (0.076 *m*) sin(48.5°) = 0.100 *N·m* **0.2%**

2R (90 *g*): $\tau_{Right2} = m_{Right2} g r_{Right2} \sin\theta_2$ = (0.090 *kg*) g (0.125 *m*) sin(65.9°) = 0.101 *N·m* **0.0%**

3R (60 *g*): $\tau_{Right3} = m_{Right3} g r_{Right3} \sin\theta_3$ = (0.060 *kg*) g (0.178 *m*) sin(73.4°) = 0.100 *N·m* **0.3%**

Part III: Balancing a Rod with Mass on one End

Ask your instructor for the metal rod with two masses at one end.

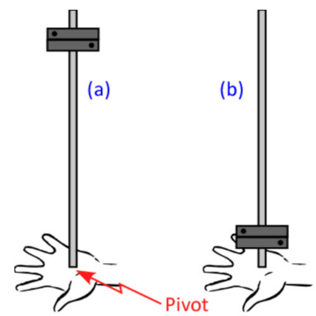

Figure 3: Balancing the rod and masses

a) *Prediction #3*: Which orientation of the rod will be easier to balance: with the masses far away from your hand (Figure 3a) or with the mass near your hand (Figure 3b).

b) Try balancing the rod each way – was your prediction correct? Draw Figure 3 in your report, and label *r* for each rod. Briefly explain the difference between the two orientations using the idea of moment of inertia, I, where

$$I = \sum_{i=1}^{N} m_i r_i^2$$

The rod is easier to balance in position (a). The moment of inertia is greater in this orientation

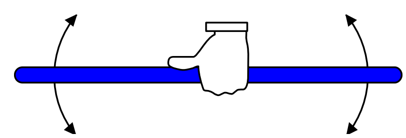

Figure 4: Twirling the plastic rods

c) You will also find a red and blue plastic rod in the lab. *Both rods are of the same mass.* Using <u>one rod at a time</u>, hold it in the center with one hand, and rotate clockwise and counterclockwise (Figure 4). Now try it with the other rod, *using the same hand as before*. Which rod is easier to start rotating? The red wand is easier to rotate than the blue. The masses are closer to the center in the red wand, so its moment of inertia is lower

d) Draw a sketch of each rod, again labeling *r* for each, and explain why in terms of the moment of inertia.

Discussion

- The actual numerical results aren't that interesting this week. With a sentence or two, summarize what you found when calculating the torque on the lever arm balance.
- Briefly define *equilibrium*. How do the left and right torques relate to the condition for equilibrium?
- Briefly discuss some sources of error when balancing the lever arm..

Friction in pivot of lever arm balance is present, but very minor (brass shaft rotates very freely); estimating when the apparatus is level is a bit tricky, as is estimating the position of the center of each loop (to measure *r*).