

Vector Addition

Fall 20xx

Solutions

Introduction

In this experiment you will use a *force table* to learn how vectors are used to represent forces, and practice adding vectors mathematically and graphically.

Theory

Forces are vector quantities, described by both their magnitude and direction. We will use masses suspended from strings to exert force on an object (a metal ring). The *resultant* is a single force calculated as the vector sum of the forces exerted on the object. We will be able to calculate the magnitude and direction of the resultant vector on the force table by examining the *equilibrant*, which is a single force that establishes equilibrium by balancing two or more forces. *Therefore, the equilibrant has the same magnitude, but opposite direction as the resultant.*

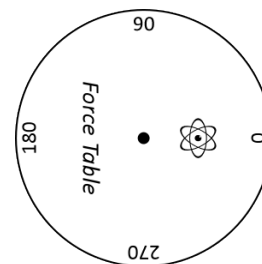


Figure 1: The force table as viewed from above

Before starting, make sure that the bubble level shows that the force table is level, and that you are looking at your force table with 0° to the right, as shown in **Figure 1** (note that there are two angle circles around the edge the force table; you will be using the inner scale).

Addition of Two Vectors

You will add two vectors $\vec{F}_1$ and $\vec{F}_2$, then calculate the magnitude and direction of the resultant, $\vec{F}$ and equilibrant, $\vec{F}_{eq}$ algebraically and graphically. First, you will measure the forces on a force table.

Vector Addition Measured on Force Table:

Set up the vectors to be added on the force table (**Figure 2**): $\vec{F}_1$ is 100 g at 0° , and $\vec{F}_2$ is 150 g at 120.0° . Note that the units of “force” are left in *grams*; this will simplify your calculations and measurements. It is not necessary to convert to Newtons!

1. Attach two pulleys to represent the forces $\vec{F}_1$ and $\vec{F}_2$ with their direction as indicated in **Figure 2**. The third pulley will represent the *equilibrant*, used to balance the other two forces, so its position can be set approximately for now. *Do not overtighten the pulley clamps!*
2. Place the ring with three strings over the center post and pass each string over a pulley. Attach a mass hanger to each string, and place additional mass on the two strings that will represent the magnitudes of $\vec{F}_1$ and $\vec{F}_2$ (note that the hangers have a mass of 50 g).
3. Estimate the angle ϕ (“phi”) as follows: Grab the third string (representing the equilibrant) and gently pull the string while moving it left and right with respect to the table. Do so until the ring is centered on the post. Set the third pulley to the position you determined and hang the string representing the equilibrant over.
4. Now place additional mass on the third string representing the equilibrant. You will recall that this force balances the resultant force of $\vec{F}_1$ and $\vec{F}_2$. Adjust ϕ and the amount of force as necessary, making sure the strings pass straight over each pulley.

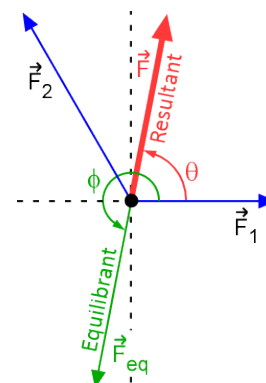


Figure 2: Adding the two vectors gives the *Resultant* and *Equilibrant* vectors

5. The system is balanced (in a state of *static equilibrium*) when the ring is centered on the central post. Create *Table I* in your journal and record your measurements of force and angle ϕ when the system is balanced.

Note on vector labels: Using a vector symbol in Excel is awkward, so you can use the old-time notation of writing vector labels in **boldface**. For example, the vector $\vec{F}_1$ can be represented in Excel as **F1**. Also note that typing this vector in plain text, F_1 implies that you are referring to the *magnitude* of the vector.

Table I:
Force Table
Measurements

Force	Magnitude (g)	ϕ (°)	$\theta = \phi - 180^\circ$ (°)
$\vec{F}_{eq}$	130	260	80

6. Calculate the ‘measured’ value of $\theta = \phi - 180^\circ$ and record in *Table I* above.
7. Write a summary (*keep it brief*) of what you did in the *Final Analysis* section of your journal.

Vector Addition by Calculation:

8. Create *Table II* in your journal and use trigonometry to calculate the magnitude of the x- and y-components of forces $\vec{F}_1$ and $\vec{F}_2$. Calculate these components directly in your journal using $\cos(\text{RADIANS}(\theta))$, where θ refers to the cell containing the angle (in degrees). See section VI (page 3) of *Useful Things to Know in Microsoft Excel*.

Table II:
Calculation of
Vector
Components

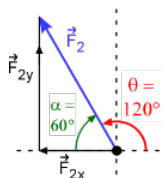
Force	Magnitude (g)	Direction, θ (°)	F_x (g)	F_y (g)
$\vec{F}_1$	100	0	100	0
$\vec{F}_2$	150	120.0	-75	129.9

9. Create *Table III* in your journal and calculate the magnitude of the resultant vector. Note that the first two columns calculate the components of the resultant vector, which are the sums (Σ , “Sigma”) of the x- and y-components, respectively, from *Table II*. Here you’ll need to use $=\text{DEGREES}(\text{ATAN}(F_y/F_x))$ in your Excel calculation to calculate the angle of the resultant vector.

Table III:
Calculation of
Resultant

ΣF_x (g)	ΣF_y (g)	$F = \sqrt{\Sigma F_x^2 + \Sigma F_y^2}$ (g)	$\theta = \tan^{-1}(\Sigma F_y / \Sigma F_x)$ (°)	$\phi = \theta + 180^\circ$ (°)
25	129.9	132.3	79.1	260

10. Finally, calculate θ , the direction of the resultant vector and $\phi = \theta + 180^\circ$, the angle of the equilibrant as measured on the force table (make sure you understand *why* you add 180° to θ to get ϕ).
11. Again, *briefly* summarize what you did here in the *Analysis* section of your journal. Be sure to include sample calculations.



Noting that angle F_2 makes with x-axis is $\alpha = 180^\circ - 120^\circ = 60^\circ$:

$$F_{2x} = -F_2 \cos \alpha = -(150) \cos 60^\circ = -75 \text{ g}$$

$$F_{2y} = F_2 \sin \alpha = (150) \sin 60^\circ = 129.9 \text{ g}$$

$$F = \sqrt{F_x^2 + F_y^2} = 132.3 \text{ g}$$

$$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right) = 79.1^\circ$$

Graphical Addition of Vectors:

12. Each member of your group will now add the vectors graphically. **Drawing on the blank side of the page, use the intersection of two darker lines near the lower left corner of a new sheet of graph paper as the origin.** Draw $\vec{F}_1$ with its scaled magnitude and direction, beginning at the origin (*Scale: 1 cm = 10 g*). *Note that the graph paper is not centimeter ruled!* You will find it easier to draw your vectors as shown in **Figure 3A** so that the arrowhead does not obscure the length of the vector:

(A) Displace the arrow from the end of the vector:



(B) Placing the arrow at the end of the vector will obscure its true length:



Figure 3: Displace the arrow from the end of the vector (A) when adding graphically

13. Now draw $\vec{F}_2$ with its scaled magnitude and direction, starting at the end of $\vec{F}_1$. **Be sure to position the protractor correctly,** as shown in **Figure 4**.

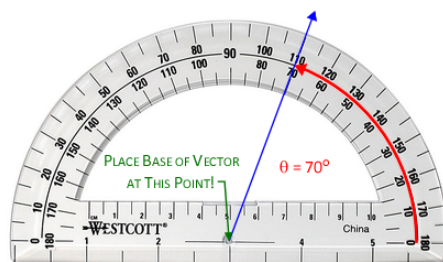


Figure 4: The correct positioning and reading of a protractor

14. Finally, draw the resultant vector, $\vec{F}$. Measure its magnitude with a ruler (in *cm*), and convert back to units of grams. Measure θ with a protractor, again making sure it is positioned correctly.
- Write your name and your lab partner's name on the top of your vector map (you will hand this map in at the end of lab).
 - Write the dimensions (in *cm and grams*) of your vectors on the graph you have drawn.
 - Label the angles that you measured.
 - Indicate the scale used on your vector map.
15. Calculate and record in your journal the % difference between the following:
- The magnitude of the force measured on the force table and its calculated value.
 - The magnitude of the force measured on the force table and measured on the graph.
 - If you find either difference to be more than a few percent, you should check your force table measurements, calculations, and the lengths of drawn vectors!
16. Again, *briefly* summarize what you did here in the *Analysis* section of your journal.

Discussion

- Begin with a summary of the three values of the magnitude (in *grams*) and direction of $\vec{F}$, and your two values of ϕ .
- Every student in this course is doing this experiment. Which method do you think gives the most consistently repeatable results when calculating the magnitude and direction of $\vec{F}$? Briefly explain why and discuss the sources of error in the other two methods.
- Why do you think the smallest mass increment provided with the force table is 5 g? Why did you not get a pile of 1 g masses? *Hint: What do we assume about pulleys?*

Vector Addition Map Larger Scale

JRm 29 Sept 2015

