Hooke's Law

Fall 2024

Introduction

Physics is the science of trying to understand why the physical world around us behaves the way it does. We do this by assuming that there is a cause-and-effect relationship among various physical quantities. We test this assumption by *mathematising* the relationship. Mathematising allows us to assess the validity of our hypothesis by comparing it to measurements made in the real world, and then to make predictions about measurements we have not yet made. For example, let's look at the relationship between the mass on a spring and the length of the spring. Does the spring get longer in direct proportion to the mass suspended? If so, we say there is a linear relationship between them. We can test this hypothesis by going to the real world (in this case the lab) and measuring the length of a spring for various masses. How do we tell from this if our hypothesis is valid? We can *plot* (graph) the length of the spring as a function of mass. If our points fall exactly on a straight line, then we conclude that our guess was a good one. Or if the points fall along a parabola, we conclude that the relationship is not linear. The relationship we deduce in this way is often called a *mathematical model*, since it is a simplified representation of the physical world.

Nature, or a mass on a spring, rarely cooperates exactly with our hypothesis. That is, the data points rarely all fall exactly on a particular line. This is because there are many other factors in any physical system, and when we choose to measure length of a spring as a function of mass, we are choosing to limit ourselves to only two of those factors and to ignore all others. Choosing what to ignore is one of the most challenging and creative aspects of physics. This is why we say that physics is a science of approximation. Our mathematical models are only approximate descriptions of complex natural systems. Even when you measure something very carefully in lab, your data will often not fall exactly on a particular curve.

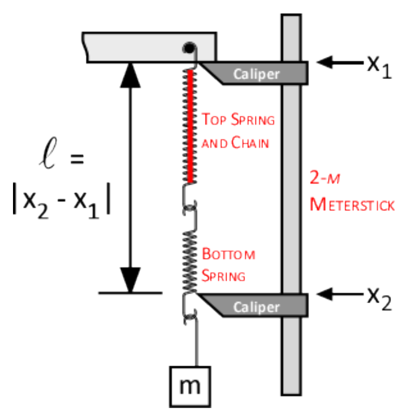
If your data do not all fall on a single line, how can you tell what the best mathematical approximation is for this physical system? This is where curve fitting comes in. You can try drawing a single straight line through your data points and see how close the line comes to the experimental points. Would the line fit better if you changed its slope or intercept a little? Would a parabola fit even better? Finding the single line or curve that comes closest to approximating the specific data points you measured in the real world can tell you which mathematical approximation best describes the relationship among the quantities you measured. If you want a better approximation, you can then include additional factors or terms. In this way we construct a mathematical model.

Purpose of This Experiment

You will study how a double-spring system stretches under the effect of a load and will test how well the theory of *Hooke's Law* predicts the behavior of your spring system. This experiment will give you practice collecting data, testing theories, and writing a lab journal (which your instructor will discuss). Today's lab journal will be graded but, as long as it is submitted on time, the grade will not count in your final lab grade. Throughout these instructions you will see references to sections of the *Introduction to Laboratory Practices* (ILP) document; refer to that document for the details.

The Apparatus

You will work with a pair of springs joined end to end. The top spring is the longer of the two, and has a chain threaded through it that limits the amount by which it can stretch. You will use a set of masses to stretch the two springs.



Theory

Hooke's law says that a spring subject to a force $\vec{F}$ will stretch an amount proportional to that force: $\ell = c \cdot F$, where c is the "compliance", a characteristic of the spring. The value of a spring's compliance is affected by its length, how tightly the spring is wound, and thickness of the metal stock used to create the spring.

Hooke's law is a simple model of the behavior of a single spring, and too simple to describe our two-spring system. So, we propose the following theory:

> *The two springs will obey Hooke's law until the upper spring has been stretched to its limit*. For any additional loads, the springs will follow Hooke's law with a different value of compliance.

We will denote the compliance of the top spring as c_1, and c_2 will represent the compliance of the bottom spring. Since the length that the two springs stretch together is the sum of the lengths that each stretch, the compliance of the two stretching together is $c_3 = c_1 + c_2$.

This model, if true, says that the total length ℓ , plotted against $\vec{F}$ will consist of two straight lines, joined by a curved transition.

Procedure

1. Begin your journal entry according to the ILP instructions about writing lab journals (ILP: "Laboratory Journal", section B "The Journal"); get used to tabulating data chronologically (ILP: "Data", section B "Numerical Data"). The idea is to measure the *total length* of both springs as a function of the masses hung.
2. Create a data table in your journal with column headers as shown below. Since you don't know how many data points you will be collecting during the experiment, *you should start the table at the top of a new page*.

m (*g*)	x_1 (*cm*)	x_2 (*cm*)	$\ell = \left\| x_2 - x_1 \right\| (cm)$

3. We ***strongly*** believe in plotting data on the graph as you take them. You first need the extreme values for the graph; maximum and minimum lengths and loads for the axes. Begin by measuring the length of the two springs combined for (a) 1500 *g* load, and (b) no load (*remove the mass hanger as well*). Length measurements should be read to the nearest 0.1 *cm*. Start a large graph in your journal, using these extreme values to set the axes, and add a few points in between (See ILP: "Graphs" for some plotting notes) *Note:* For this experiment, it is not necessary to include the region between zero and your shortest ℓ on the graph.
4. Use your graph to decide how much mass to hang next. Generally, take less data where the function is predictable and more where it changes rapidly (*This is why we say to plot as you go along!*) Take as much data as is necessary to establish the entire plot, repeating points that seem 'off'; it doesn't matter what order you collect your measurements after the first two points. Pay particular attention to the region of the graph where the springs are lightly loaded (≤ 200 *g*); you may discover something interesting!
5. The graph should conform to the rules in the ILP, "Graphs" section. It must have:
   - a descriptive title, like *"The length of two springs vs. the mass hung from it"*;
   - labels and units on both axes, e.g. *"M_{hung} (g)"*
   - the appropriate symbol used to plot data points: ⊙

   You may have noticed that you are plotting mass (units: *grams*), not force (units: *Newton*). However, since we haven't discussed gravity yet in class, you may measure 'force' in units of grams.
6. Calculate spring compliances (*the slopes*) from the graph: The larger slope is $c_3 = c_1 + c_2$ (both springs); the smaller slope is c_2 (the bottom spring). Use a ruler to extend the lines of fit across the graph. Mark slope points with a small 'x', picking points near opposite ends of the extrapolated line (*don't use data points!*) to calculate the slope. *Be sure that you fit only the **linear** regions of the data points; do not try to incorporate the non-linear regions in your fit!*
7. Calculate c_1, the compliance of the top spring, by subtraction.

Discussion

- Begin with the **numerical results** (c_1 c_2, and c_3) and a **reference to your graph** with a verbal description of its general shape.
- Your journal should always include a statement about measurement uncertainty. Think about your measurements during the experiment. Did you find anything particularly difficult to deal with? Did you have to estimate any quantities? *Note that your inability to measure lengths down to the atomic level is* *<u>not</u>* *considered a measurement error!*
- Now discuss whether or not your results are consistent with the theory proposed for the double-spring system (*page 2*). Today this will probably mean that there will be two ranges where (by inspection of the graph) the springs are consistent with Hooke's law and one where they aren't. In addition, you might notice some peculiar behavior when the spring system is lightly loaded – if you read step 4 carefully. In your discussion, you should note where there is agreement, and attempt a brief conjecture about why there is disagreement.

Please clean up your work area when you have completed your experiment. Return the setup to the condition you found it when you entered the lab. Be sure to read through the "Laboratory Etiquette" section of the *Laboratory Syllabus* before leaving. And be sure to re-read the *Introduction to Laboratory Practices*, as well as the sample lab journals, before next week's lab.