

Let's look at some of the most useful/common ones. In what follows,  $V$  = option value,  $S$  = stock price, and  $\tau = T - t$  = time to maturity (where  $t$  = current time).

(1) Delta.  $\Delta = \frac{\partial V}{\partial S}$

This is the sensitivity of the option's value to the stock price. This is one of the most basic measures of option price sensitivity. Another reason it's used a lot is that

$|\Delta|$  is a decent approximation for the

MONEYNESS

You're so money and you don't even know it

of an option (i.e. the probability that the option expires "in-the-money").

Note that moneyness is actually  $\frac{\partial V}{\partial K}$ , where  $K$  is the strike price, but since we compute  $\Delta$  anyway, it's good to know it can be used to give us a sense of moneyness.

This means that  $0 \leq |\Delta| \leq 1$ , by the way (or else it can be a terrible approximation for a probability). [In fact,  $-1 \leq \Delta \leq 0$  for a European put &  $0 \leq \Delta \leq 1$  for a European call.

We can show  $\Delta(\text{call}) - \Delta(\text{put}) = 1$  for a call and put with the same strike & maturity.]

(2) Vega  $\nu = \frac{\partial V}{\partial \sigma}$

OK, a couple of things. One - vega isn't a Greek letter. Two -  $\nu$  is the letter nu. Three - sometimes vega is represented by  $\kappa$ , which is the letter kappa. Why?? Who knows. Maybe because "Vega" sounds cooler.

Either way, vega is the sensitivity of the option's value to changes in the volatility. (Remember that  $\sigma$  is the standard deviation of the return of the stock.)

This measure is especially important in volatile markets.

(3) Theta  $\theta = -\frac{\partial V}{\partial \tau}$  aka "time decay"

$\tau$  is the number of years between now and maturity, so  $\theta$  measures the change of the option's value over time. (We usually divide by 365 to get the daily loss of value per share.)

Theta is giving us the change in the TIME VALUE of the option — that is, how

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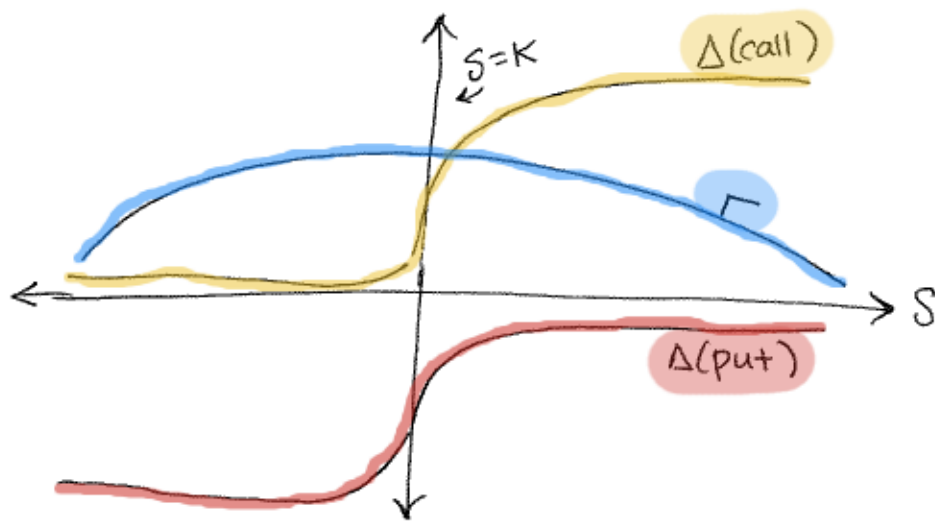
the decreasing time to maturity impacts the value. The idea here is that if your option is worth, say, nothing now, it still has a shot at expiring in-the-money because there's time remaining for the stock price to move in the right direction. But as the time to maturity goes down, you have less hope of your option gaining value (because less time for stock to go in the right direction).

So theta is almost always negative. One exception to this rule of thumb: deep in-the-money put option (i.e. k is well above the stock price).

(4) Gamma  $\Gamma = \frac{\partial \Delta}{\partial S} = \frac{\partial^2 V}{\partial S^2}$

$\Gamma$  tells us the sensitivity of  $\Delta$  to changes in the stock price. What this means, practically speaking, is that  $\Gamma$  tells us the convexity of the value, written as a function of the stock price (with all other variables held constant).

The relationship between  $S$ ,  $\Gamma$ , and  $\Delta$  can be summarized like this:



How to interpret this graph: when the stock is at the money,  $S=K$ . As the stock price rises or falls from there,  $\Gamma$  falls, while  $\Delta$  always rises with the stock price only.

The gap (vertically) between  $\Delta(\text{call})$  &  $\Delta(\text{put})$  is of height 1 everywhere.

$\Gamma$  is useful for HEDGING. (Neutralizing  $\Gamma$  makes your hedge effective over wider range of values.)

(5) 
$$\text{Vanna} = \frac{\partial \Delta}{\partial \sigma} = \frac{\partial^2 V}{\partial S \partial \sigma} = \frac{\partial \overset{\leftarrow \text{vega}}{V}}{\partial S}$$

Vanna, aka "Dvega Dspot" and "Dvega Dvol", is another one of these sounds-like-a-greek-letter-I-guess terms. This is the last one we'll define, and like  $\Gamma$ , it's also useful for hedging: as the volatility

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changes, vanna tells a trader the impact on a delta-hedge.

Some other fun ones we won't go into (because they're not used as frequently) are vomma, veta, vera (aka rho), zomma, ultima, charm, color, speed, ... yeah.

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So now we have a little bit of an idea of some things that are done in the finance world — and more importantly, we can understand where all of that comes from!

So if you go off into finance from here, you are in a good place to understand & develop things in the Black-Scholes context. And if you are more interested in the math behind it all rather than applications, I think we can all agree we've seen plenty of that, too! 😊

Good luck on finals & in your futures! ❤️

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