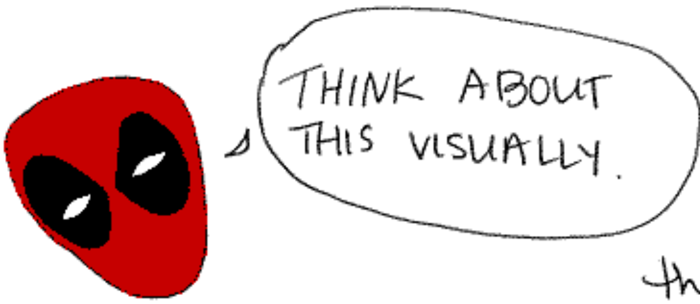
Interlude on Convexity

Let $I \subseteq \mathbb{R}$ be an interval in $\mathbb{R}$.

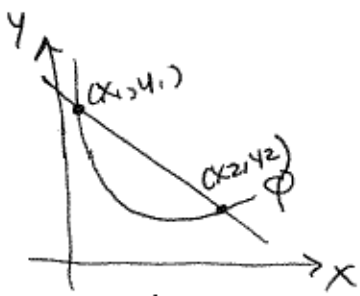
Def. $\varphi : I \to \mathbb{R}$ is a convex function if

$$\varphi(t x_1 + (1-t) x_2) \leq t\varphi(x_1) + (1-t)\varphi(x_2)$$

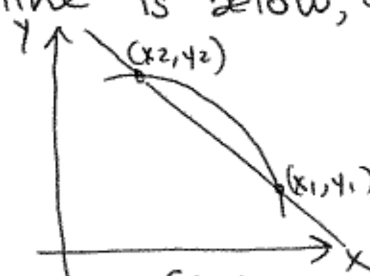
$\forall\ x_1, x_2 \in I,\quad t \in [0,1]$.



→ Take 2 points on the curve of φ and connect them with a straight line. If the line is above the curve, φ is convex. If the line is below, φ is concave.



Convex.
aka "concave up"



Concave.

Jensen's Inequality. Let $(\Omega, \mathbb{P})$ be a finite probability space. Let $Y : \Omega \to \mathbb{R}$ be a random variable. Let $\varphi : I \to \mathbb{R}$ be a convex function and assume $Y(\omega) \in I$ $\forall\, \omega \in \Omega$. Then

$$\mathbb{E}[\varphi(Y)] \geq \varphi(\mathbb{E}[Y])$$