Historical Uses of Game Theory in Battles during the World War II

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Abstract

In 1954, O.G. Haywood used game theory to analyze the military decisions used in the Battle of the Bismarck Sea, a battle fought during the World War II. Haywood analyzed the Battle of the Bismarck Sea by using a two-person zero-sum game [1]. This paper discusses the fundamental concepts of the two-person zero-sum game and some Nash Equilibrium dominance ideas as well as the strategies applied to the Battle of the Bismarck Sea based on the actual military operation.

1 Introduction

Game theory is the science of strategic reasoning, in such a way that it studies the behavior between intelligent rational decision-makers. Game theory is usually applied to economics, psychology, and political science. However, this paper will focus on the use of game theory in the military area, a field that is not typically well known by people. It was quite hard to find resources at the beginning, as not much research had been conducted in this field. But Haywood provided a solid analysis of applying game theory strategies in the military decision process during the World War II in his paper, *Military Decision and Game Theory*. In the paper, Haywood focused on analyzing the use of game theory in a typical battle, the Battle of the Bismarck Sea.

The Battle of the Bismarck Sea was a battle fought in February 1943, during the World War II in Southeast Asia, between the Japanese Navy and the U.S. Air Force [1]. General Kenney was the commander of the U.S. Air Force and Admiral Imamura was the commander of the Japanese Navy [2]. The Japanese Admiral ordered to deliver reinforcements to Japanese
soldiers fighting in Papua New Guinea. The struggle for Papua New Guinea had reached a critical stage (see Figure.1) [1]. The Japanese were forced to make a choice between two available routes —either the Northern route, through the Bismarck Sea, or the Southern route, through the Solomon sea.

Figure 1: Red area was under Japanese control and blue area was under U.S. control [4]

General Kenney knew all of the routes available to his enemy [1]. If he predicted Japanese’s move correctly and sent his planes toward the route, then the U.S. Army would have more days to bomb the Japanese. The “payoffs” in this game were the number of days the U.S. had to bomb. Since the number of days was the same for each country, both U.S. and Japanese had same amount of payoffs except we use positive number to represent the payoff for U.S. and use negative number to show the payoff for the Japanese. Therefore, this was a two-person zero-sum game. In order to further analyze the possible actions and to compare outcomes for these two countries, we shall acquire some basic understanding of the two-person zero-sum game.

2 Preliminaries of Two-Person Zero-Sum Games

As mentioned, we will use the approach of Game Theory, especially the Two-Person Zero-Sum Game to analyze the Battle of the Bismark Sea. A two-person zero-sum game represents the situation that one person’s gain is exactly balanced by the other person’s loss [8]. Thus, we will get a sum of zero when adding up the total gains and subtracting the total lose. In
order to analyze a two-person zero-sum game, we introduce a simple mathematical description of a game, the strategic form [8].

**Definition 2.1.** The **strategic form** of a two-person zero-sum game is given by a triplet $(X, Y, A)$, where

1. $X$ is a nonempty set, the set of strategies of Player I
2. $Y$ is a nonempty set, the set of strategies of Player II
3. $A$ is a real-valued function defined on $X \times Y$ [8]. (Thus, $A(x, y)$ is a real number for every $x \in X$ and every $y \in Y$.)

This definition could be addressed as follows. Player I chose $x \in X$ and Player II chose $y \in Y$ concurrently, and each player was oblivious of the choice of the other. The amount Player I could win from Player II is $A(x, y)$. If $A < 0$, Player I loses this amount. Therefore, $A(x, y)$ provides the winnings of Player I and the losses of Player II.

The maximin value and the maximin value are frequently used terminologies in two-person zero-sum game. As they will be used in later section, we give the definitions as follows.

**Definition 2.2.** The **maximin value** of a player is the largest value that the player can be sure to get without knowing the actions of the other players [8].

**Definition 2.3.** The **minimax value** of a player is the smallest value that the other players can force the player to receive, without knowing his actions [8].

Under many situations we will obtain the same maximin and minimax values. When such encounter comes into existence, we call this outcome an equilibrium outcome and we obtain a saddle point [9].

**Definition 2.4.** When the maximin and minimax are said to be in equilibrium, the outcome associated with them is called a **saddle point** [9].

In the next section, we will talk about how two-person zero-sum games with a saddle point can apply to the Battle of the Bismarck Sea. As for the variations of the two-person zero-sum game, such as zero-sum games without a saddle point, we will consider them to be possible future work directions.
3 Analysis of the Battle of the Bismarck Sea

According to the United States military doctrine of decision, a military commander makes decision either based on enemy capabilities or on enemy intentions [6]. In the Battle of the Bismarck Sea, the decision made by General Kenney was based on the enemy capabilities, that is, what Japanese Admiral was able to do to oppose him. General Kenney used a five-step process called Estimate of the Situation to make decision [9].

Step 1. The mission

As ordered by the Supreme Commander General MacArthur, the mission for General Kenney was to cut off and expose maximum annihilation on the Japanese reinforcements convoy [1].

Step 2. Situation and Courses of Action

As provided in the introduction section, the general situation was commonly known. However, there was a deciding factor pointed out by General Kenney’s staff. The bad weather was predicted to appear in the north, accompanied by the poor visibility. In addition, the poor visibility in the north would directly limit the number of bombing days for the U.S. to only two days, while for the south the good weather and visibility would ensure more days of bombing [1].

For Japanese troop, no matter which route its commander choose, it would take three days to get to the final destination.

Step 3. Analysis of the Opposing Courses of Action

Since each commander would have two alternative choices, there were a total of four possible clashes that could potentially arise.

In the first scenario, General Kenney would concentrate most of his aircraft along the Northern route and the Japanese Navy would also take the Northern route. As mentioned earlier, there would be a total of two days of bombing due to the poor visibility (Figure 2(a)) [4].

In the second scenario, General Kenney would concentrate most of his aircraft along the Northern route again, but this time, Japanese Navy would take the Southern route. Since the reconnaissance was limited along the Southern route, the Japanese convoy could be missing
during the first day, allowing only two days of bombing (Figure 2(b)) [4].

In the third scenario, the U.S. Air Force would be located along the Southern route, and the Japanese Navy would take the Northern route. Due to the poor visibility and the low-level of reconnaissance, the Japanese would be missed for two days, allowing for only one day of bombing (Figure 2(c)) [4].

In the last scenario, both the U.S. Air Force and the Japanese Navy would take the Southern route. Due to the good visibility and the majority of air force, General Kenney could have three days of bombing (Figure 2(d)) [4].

(a) Scenario 1 (b) Scenario 2 (c) Scenario 3 (d) Scenario 4

Figure 2: Possible battles [4]

Step 4. Comparison of Available Courses of Action

General Kenney sought a conflict that provided him with the maximum time of bombing, while the Japanese Admiral desired the minimum exposure to bombing [2]. But neither Kenney nor Admiral could determine the result of the battle based on his own decision. All possible conflicts in this battle were converted into the following matrix form. The rows listed Kenney’s strategies, and the columns listed the Japanese strategies. The number at each intersection represents the number of days of bombing (Figure 3).

<table>
<thead>
<tr>
<th>Kenney strategies</th>
<th>Japanese strategies</th>
<th>$1$-Northern route</th>
<th>$2$-Southern route</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1$-Northern route</td>
<td></td>
<td>2 days</td>
<td>2 days</td>
</tr>
<tr>
<td>$2$-Southern route</td>
<td></td>
<td>1 day</td>
<td>3 days</td>
</tr>
</tbody>
</table>

Figure 3: Matrix representation of the Battle of the Bismarck Sea [1]

When it comes to game theory, we add an extra row and an extra column (Figure 4).
For Kenney, he wanted to ensure his outcome to be greater than or equal to the minimum number in any row so he acquired as many days of bombing as possible. Thus, we put the minimum values of each row into that extra column. In this matrix, we can easily see that if Kenney stayed with the Northern route, he guaranteed two days of bombing. If Kenney stayed with the Southern route, he may face a situation that only allowed one day of bombing. Based on the doctrine that a commander makes his decision on the estimation of the enemy’s capability, Kenney would definitely select to search the Northern route, as it gave the greatest promise of success [3]. Therefore, Kenney would select the maximum value within the column of minimums, that is, the maximin value —2.

As for Admiral, he wanted to minimize his exposure to bombing, so that he would note the worst situation that would occur by checking numbers in each column. Thus, we put the maximum values of each column in that extra row. In the matrix, we can see that if Admiral stayed with the Northern route, he would have a max of two days of bombing. But if he stayed with the Southern route, he might be exposed to a three-day bombing. Therefore, Admiral would select the minimum value within the row of maximums, that is, the minimax value —2.

![Figure 4: Strategic form representation of the Battle of the Bismarck Sea [1]](image)

<table>
<thead>
<tr>
<th></th>
<th>Japanese strategies</th>
<th>Minimum of row</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Kenney strategies</strong></td>
<td><strong>$s1$</strong></td>
<td><strong>$s2$</strong></td>
</tr>
<tr>
<td>$s1$</td>
<td>2 days</td>
<td>2 days</td>
</tr>
<tr>
<td>$s2$</td>
<td>1 day</td>
<td>3 days</td>
</tr>
<tr>
<td><strong>Maximum of column</strong></td>
<td>2 days (minimax)</td>
<td>3 days</td>
</tr>
</tbody>
</table>

*Step 5. The Decision*

Base on our analysis from the perspective of game theory, both General Kenney and Japanese Admiral would choose the Northern route. Not surprisingly, the outcome we predicted was identical to the actual result. In the Battle of the Bismarck Sea, General Kenney finally decided to put his reconnaissance along the Northern route. The identical results was not a coincidence. Recall that in game theory this outcome is called a saddle point, which means both players yield same outcome. There are two reasons why players in a zero-sum game should choose a strategy associated with a saddle point [9]. First, a player’s security
level will be maximized by choosing the saddle point; second, a player will keep the other player from maximizing his security level by choosing the saddle point.

The concept of saddle point leads to the discussion of the role that Nash Equilibrium played in the Battle of the Bismarck Sea. When it comes to Nash Equilibrium, we always want to find strategy for players that neither player has an incentive to change strategy based on what the other player does. Notice that even though only a few games have a dominant-strategy equilibrium, dominance could still be useful [5]. In the Battle of the Bismarck Sea, neither Kenney nor Admiral has a dominant strategy [5]. The payoff table in Figure 3 shows that Kenney would have chosen the Northern route if he thought Admiral would choose the Northern route, also, if he thought Admiral would have chosen the Southern route. On Admiral’s side, he would have chosen North route if he thought Kenney would choose the Southern route, and he would be indifferent between choices if he thought Kenney would choose the Northern route. However, we can use the concept of "weak dominance" to find a possible equilibrium.

*Strategy* $s'_i$ is *weakly dominated* if there exists some other strategy $s''_i$ for player $i$ which is possibly better and never worse, yielding a higher payoff in some strategy profile and never yielding a lower payoff [5].

One way to acquire weak-dominance equilibrium is through deleting all the weakly dominated strategies of each player [5]. In the Battle of the Bismarck Sea, Admiral’s choice of go south is weakly dominated by the choice of going north, because his payoff of going north was never smaller than his payoff of going south. But there was no weakly dominated strategy for Kenney. Hence, we have to go to further discussion of the idea of the iterated-dominance equilibrium.

An *iterated-dominance equilibrium* is a strategy profile found by deleting a weakly dominated strategy from the strategy set of one of the players, recalculating to find which remaining strategies are weakly dominated, deleting one of them, and continuing the process until only one strategy remains for each player [5].

Applied to the Battle of the Bismarck Sea, the iterated-dominance equilibrium implies that Kenney believes that Admiral will choose the North route since it is weakly dominant [5]. After taking one scenario out of consideration, Kenney has a strongly dominant strategy: to go north so that he would achieve payoffs strictly greater than results if he would go south.
Therefore, the strategy \((North, North)\) was an iterated-dominance equilibrium, which was the actual outcome of the Battle.

4 Conclusion and Future Work

Throughout this paper, we analyzed military decisions of the Battle of the Bismarck Sea between Japan and U.S. during the World War II based on the idea of two-person zero-sum game and Nash Equilibrium. The strategies used in the Battle of the Bismarck Sea offered an intuitive example of the use of game theory in historical wars. Numerous other wars have attracted researchers from various academic fields. The Cold War was one of those that not only applied game theory on an aspect of pure mathematics but also involved international relations and economics [7]. Actually, the use of game theory in today’s military decision-making process is very similar to what we see in the Cold War. As military decisions should be made depending on both the strength of a nation’s army and its political power, the explicitly use of game theory in military decision-making process has been made to demonstrate understanding of problems in international relations and thus enlightens military officer on decision-making [7]. Since game theory plays an important role in providing both mathematical solutions and political inspiration, studying it could shed light on future political and military movements.

According to my current research, one possible future work direction is to modify the order of play in the Battle of the Bismarck Sea. For instance, if either Kenney or Admiral moved first rather than simultaneously move, then the outcomes may have been different. It is for sure that the history could not be rewritten, but studying it will help us to better understand decisions in wars from the perspective of game theory and provides us with valuable information for possible future military encounters. Another possible future work direction is to look at a variation of the two-person zero-sum game, that is, the two-person zero-sum game without a saddle point. In the discussion above, we only talked about the two-person zero-sum game with a saddle point. However, there exists such situation where no saddle point can be found. Consider the Battle of Avranches-Gap, which occurred in August 1944 right after the invasion of Normandy [9]. This battle can be regarded as a two-person zero-sum game without a saddle point as there does not exist an equilibrium outcome. In other words, it must be the case that
either one player can do better if one can deduce the enemy’s intention [9]. This variation of
the two-person zero-game is worth further exploration as it not only has been used in previous
battles but it may also benefits future military decision making process.

References

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York: Duell, Sloan and Pearce, 1949.


