Capture Time in Variants of Cops & Robbers Games

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Thesis Defense
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- Graphs on which a cop can win (i.e. capture) in finite time are called **cop-win**.
- Game takes no more than $n-4$ moves on cop-win graphs with $n \geq 7$ [1, 3]. (Note: original game can't take more than n^2 on any graph, including directed graphs.)

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Theorem

On a connected, undirected, simple graph on n vertices, a cop can capture a drunk in expected time $n+o(n)$.

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[Spoiler: it isn't.]

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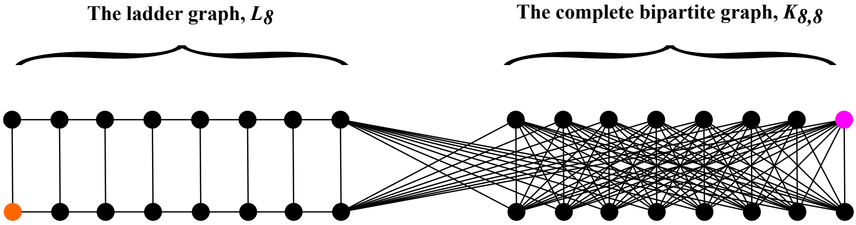
Example. "Ladder to the Basement"



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- So if cop and drunk start d apart, takes $4(4n^{2/3})(d-3)$ moves on average to get down to distance 3.
- After that, greedy algorithm only takes Δ more steps on average ($\Delta =$ highest degree).

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Expected distance after Stage 1 is $O^*(\sqrt{n})$.

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- This is a corollary of the Varopoulos-Carne bound [6, 8]:

Theorem

Let $P = (p(x,y))_{x,y \in V(G)}$ *be the transition matrix associated with a simple random walk* $\{x_0, x_1, \ldots\}$ *on* G. *Then*

$$p^t(x,y) \le \sqrt{e}\sqrt{\frac{\deg(y)}{\deg(x)}} \exp\left(-\frac{(d(x,y)-1)^2}{2(t-1)}\right)$$

where $p^t(x,y) = \mathbb{P}(x_t = y | x_0 = x)$.

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- Graph theory fun fact: $\operatorname{diam}(G) + \Delta \leq n + 1$.
- So total time is at most $n + o(n)$.

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  - Expected capture time is at least $\frac{n+1}{2}$ on any graph using DFS.

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Bonus: this remains true whether the cop gets to choose her initial position or not.

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Theorem

A graph is hunter-win if and only if it is a lobster.

Characterization

Definition

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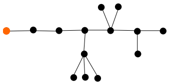
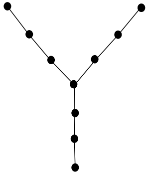


A lobster.



Not a lobster.

Characterization

Lemma

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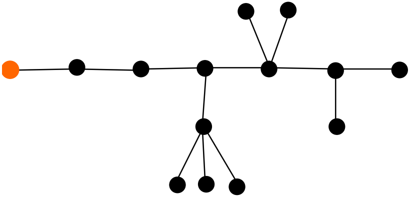

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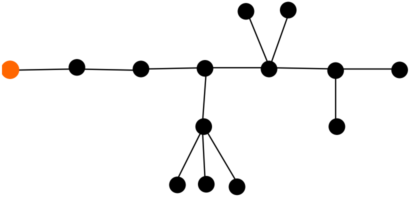

Define an **odd** (resp. **even**) **mole** to be a mole who starts at an odd (resp. even) distance from the marked vertex.

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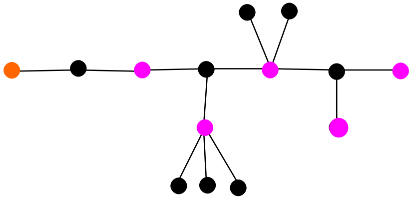

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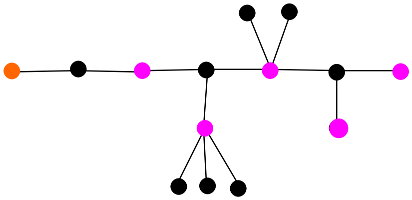
Proof by picture. After hunter's 1^{st} step:



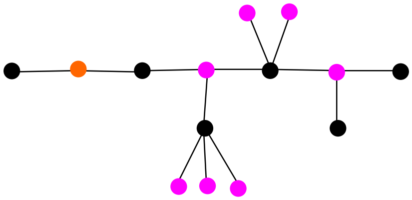
Orange vertex = hunter's position
Purple vertex = even mole's possible positions

Characterization

Lemma

Lobsters are hunter-win.

Proof by picture. After hunter's 2^{nd} step:



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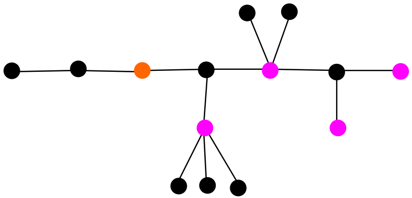
Characterization

Lemma

Lobsters are hunter-win.

Proof by picture. After hunter's 3^{rd} step:



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Characterization

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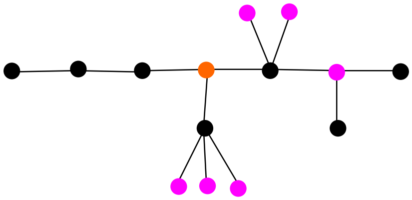
Lobsters are hunter-win.

Proof by picture. After hunter's 4^{th} step:



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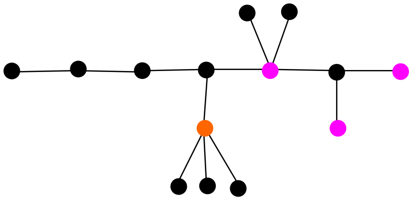
Lobsters are hunter-win.

Proof by picture. After hunter's 5^{th} step:



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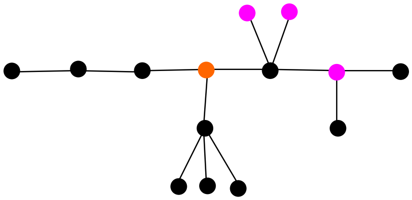
Characterization

Lemma

Lobsters are hunter-win.

Proof by picture. After hunter's 6^{th} step:



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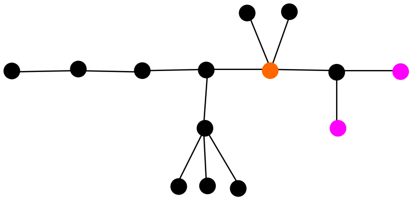
Characterization

Lemma

Lobsters are hunter-win.

Proof by picture. After hunter's 7^{th} step:



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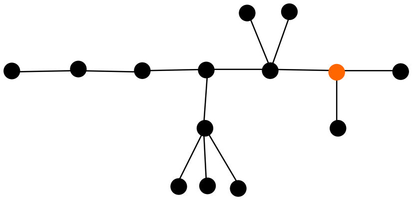
Proof by picture. After hunter's 8^{th} step:



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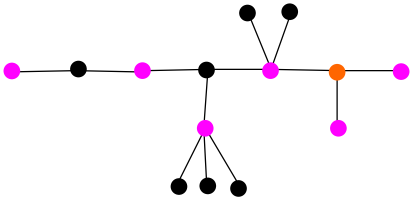

Orange vertex = hunter's position
Purple vertex = odd mole's possible positions

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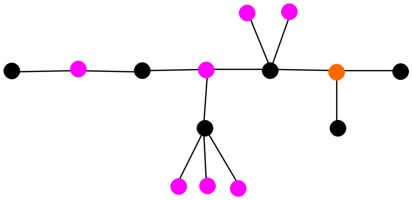
Proof by picture. After hunter's 9^{th} step:



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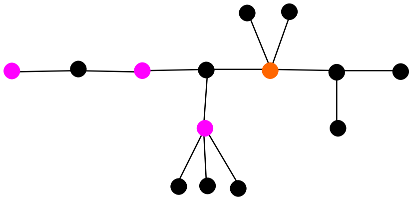
Proof by picture. After hunter's 10^{th} step:



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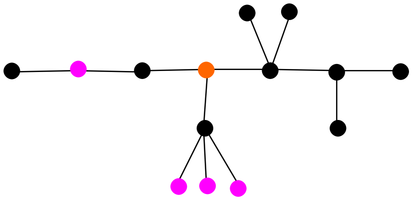
Proof by picture. After hunter's 11^{th} step:



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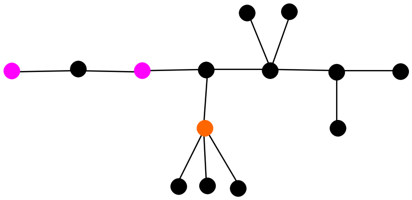
Lobsters are hunter-win.

Proof by picture. After hunter's 12^{th} step:



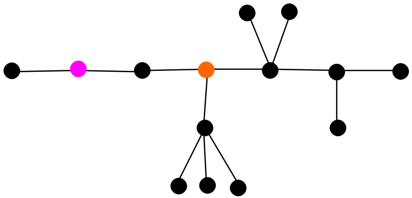
Orange vertex = hunter's position
Purple vertex = odd mole's possible positions

Characterization

Lemma

Lobsters are hunter-win.

Proof by picture. After hunter's 13^{th} step:



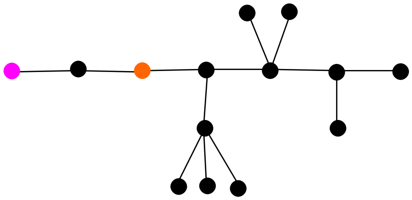
Orange vertex = hunter's position
Purple vertex = odd mole's possible positions

Characterization

Lemma

Lobsters are hunter-win.

Proof by picture. After hunter's 14^{th} step:



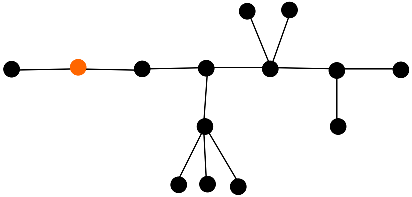
Orange vertex = hunter's position
Purple vertex = odd mole's possible positions

Characterization

Lemma

Lobsters are hunter-win.

Proof by picture. After hunter's 15^{th} step:



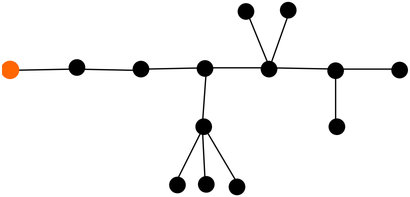
Orange vertex = hunter's position
Purple vertex = odd mole's possible positions

Characterization

Lemma

Lobsters are hunter-win.

Proof by picture.



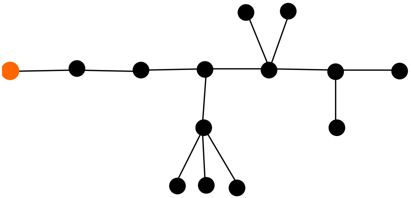
This is an optimal strategy for the hunter, by the way.

Characterization

Lemma

Lobsters are hunter-win.

Proof by picture.

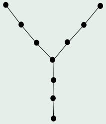


Q.E.D.!

Characterization

Lemma

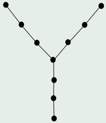
A graph G is a lobster if and only if it is a tree that doesn't contain the three-legged spider:



Characterization

Lemma

A graph G is a lobster if and only if it is a tree that doesn't contain the three-legged spider:



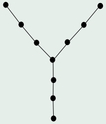
Lemma

A graph G is mole-win if:

Characterization

Lemma

A graph G is a lobster if and only if it is a tree that doesn't contain the three-legged spider:



Lemma

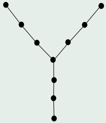
A graph G is mole-win if:

- *G is the three-legged spider.*

Characterization

Lemma

A graph G is a lobster if and only if it is a tree that doesn't contain the three-legged spider:



Lemma

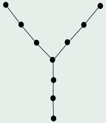
A graph G is mole-win if:

- *G is the three-legged spider.*
- *G is the cycle C_n.*

Characterization

Lemma

A graph G is a lobster if and only if it is a tree that doesn't contain the three-legged spider:



Lemma

A graph G is mole-win if:

- *G is the three-legged spider.*
- *G is the cycle C_n.*
- *G contains a mole-win subgraph.*

References

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Thank you!

Table of Contents

Lemma

Let G be any graph and let $x_0 \in V(G)$ be any vertex in G. Let $\{x_0, x_1, x_2, \ldots\}$ be any random walk on G beginning at x_0. Then $\mathbb{P}(d(x_0, x_4) < 4) \geq 1/s$, where $s = 4n^{2/3}$.

Lemma

Expected distance after Stage 1 is less than $1+\sqrt{n(1+5\log n)}$.

Lemma

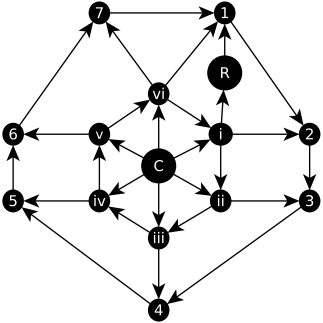
Expected distance after Stage 2 is less than $(5\log n)^{3/4}n^{1/4}$.

A quadratic digraph

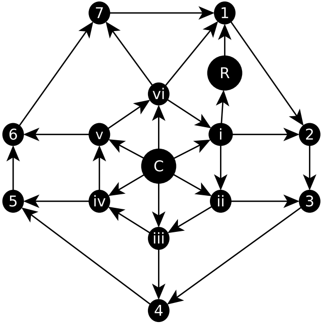
Define a $R(k)$ to be a reflexive directed graph on $n = 2k+1$ vertices consisting of:

- an "outer ring" comprised of a (counterclockwise)-directed k-cycle
- an "inner ring" comprised of a (counterclockwise)-directed $(k-1)$-cycle
- arcs from a vertex in the inner ring to a vertex in the outer ring configured such that $k-2$ vertices in the inner ring are incident with one such arc, and 1 vertex in the inner ring is incident with two such arcs
- an "internal vertex" (C) that is out-directed to every vertex in the inner ring
- an "external vertex" (R) incident with two arcs

The graph $R(7)$



The graph $R(7)$



Lemma

$R(k)$ is cop-win for all k and the capture time is $\Theta(n^2)$.