Bridge to Higher Mathematics

Sam Vandervelde

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This book was produced using the LaTeX typesetting system.

The cover artwork was designed by Eunice Cheung. The image depicts a footbridge at the Olbrich Botanical Gardens in Madison, Wisconsin.

Soli Deo gloria

Contents

To the Student

Mathematics is a wonderful subject. Consequently, a mathematics course can and should be a rewarding experience. My primary purpose in writing this text was to create a book to accompany such a course; a book that would convey the splendid nature of mathematics while presenting a few topics within the subject in as clear a manner as possible. The book is designed to draw the reader into the subject; to encourage the student to participate in the development of the ideas. For mathematics is not a spectator sport—one must actively engage in the material to properly understand and enjoy it.

This book is intended for students who have completed a standard high school mathematics curriculum and have discovered in the process that they enjoy the subject and wish to pursue more advanced studies such as group theory or real analysis. The reader has presumably also taken a calculus course; however, the text does not assume any knowledge of calculus. This was done so that the material would also be accessible to high school students and so that a course based on this text could be taken concurrently with a calculus course. The examples and problems assume a familiarity with algebra, properties of common functions such as $\cos x$ or e^x, elementary Euclidean geometry, and sets of numbers such as the integers or real numbers.

From a pedagogical standpoint, the goal of this text is to equip students with the tools and background necessary to succeed in upper level math courses. In particular, a course utilizing this book will aim to develop a student's capacity for mathematical reasoning and ability to write a sound mathematical proof. Once an interesting result has been discovered (or assigned for homework), one is faced with the challenge of explaining why it is true. There is a standard set of tools, strategies and vocabulary for accomplishing such a task. For this reason the third chapter serves as the pivot point around which the remainder of the book is set. The first two chapters introduce the framework of logic and set theory necessary for the discussion of proof techniques that comes next. The subsequent four chapters then provide an opportunity to employ and practice these techniques while laying the foundation for further mathematical study.

There are a number of features sprinkled throughout the text that warrant a brief explanation. For starters, at regular intervals the reader will encounter elementary questions following new definitions or techniques. These are dubbed "Concept Checks" and look like this:

a) How is it possible that there is an interstate highway in Hawaii?

The reader is strongly encouraged to mentally answer these questions right away, in order to help cement new concepts in place as soon as they are introduced. Answers to all the Concept Checks within each section may be found at the conclusion of that section. Somewhat less frequently the reader will be asked to supply a step in an argument or otherwise participate in the development of an idea via a "Quick Query" such as

b) How can one most effectively utilize the answers to all the exercises included at the back of the book?

Answers to the Quick Queries are also located at the end of each section.

Another feature that will quickly become apparent are the Mathematical Outings. These self-contained activities are intended for group investigation during class time or as an entertaining exploration for individuals to undertake to complement the material presented in that section. Their purpose is to help reinforce particular concepts as well as to provide a regular reminder that mathematics is a delightful, intriguing subject. The wide boxes appearing on almost every page hardly need an explanation—they highlight important definitions and techniques within each section. Finally, there is a special reference section included at the end of each chapter which summarizes all the vocabulary, concepts, techniques, and proof strategies found in that chapter. These sections are meant to aid in review and serve as a resource while studying later chapters. Complete sample proofs also appear in the reference section. Results in the text for which a sample proof is available are indicated by a dagger. (†)

Hundreds and hundreds of creative, stimulating, accessible questions have been composed for this book. In some sense, a text is only as strong as its selection of questions, since it is through working on these questions that a student internalizes the ideas laid out on the preceding pages. The questions are grouped into exercises, which need only an answer or a brief explanation, and writing problems, which require a proof: a complete mathematical explanation written in full sentences. Answers to all the exercises and hints to all the writing problems are given in Chapter 8, along with a few thoughts on how to best take advantage of them.

I hope you enjoy the book. And now, on to the mathematics.

Sam Vandervelde
St. Lawrence University
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