Pressure and Buoyant Force

Introduction

In this experiment, you will use the pressure–depth relation to calculate the pressure at some depth below a water surface, and you will use Archimedes' principle to calculate the buoyant force acting on an object.

Part I. The Pressure–Depth Relation:

Theory

A body, which is less dense than water, placed on a water surface will sink into the liquid until the body experiences a buoyant force, \( \overline{F}_b \), that equals its weight, \( \overline{W} \). This means that when the body floats, its weight and the buoyant force are the same in magnitude but opposite in direction (sound familiar?). You will use a cylinder (an aluminum can) so that the buoyant force due to the fluid acts only on the bottom of the cylinder if the can floats vertically.

Once you know the force which acts on the bottom of the can and the area of the bottom you can find the pressure on the bottom of the can. This is a gauge pressure because it assumes that the downward force is due only to the weight of the can and that the atmosphere makes no contribution.

From the definition of pressure, we have:

\[
\text{Pressure} = \frac{\text{force}}{\text{area}} = \frac{F_b}{A} = \frac{W}{A} = \frac{mg}{A}
\]

\[
\text{units: } \frac{N}{m^2} = \text{Pascal}
\]

This gauge pressure is the pressure of the water on the cylinder bottom at that depth below the surface.

We can also use the pressure-depth relation to calculate the pressure some distance below the fluid surface: \( P = \rho_w gh \), where \( \rho_w \) (the Greek letter “ro”) is the density of water (998.0 kg/m\(^3\)), and \( h \) is the depth in the liquid.

Experiment #1:

1. Create a data table in your report with the following headers:

<table>
<thead>
<tr>
<th>Can (Large/small)</th>
<th>depth, ( h ) (m)</th>
<th>can diameter, ( D ) (m)</th>
<th>can bottom area, ( A ) (m(^2))</th>
<th>mass (can+sand), ( m ) (kg)</th>
<th>weight (can+sand), ( W ) (N)</th>
<th>( P_1 = \frac{W}{A} ) (Pa)</th>
<th>( P_2 = \rho_w gh ) (Pa)</th>
<th>%Diff ( P_1 \ &amp; \ P_2 )</th>
</tr>
</thead>
</table>

2. Place a piece of tape on the small can about ¾ of the distance from the bottom, and draw a line on the tape. Load the can with sand so that it floats upright and level at this line (be careful not to sink your can!) Tilt the can to allow any air trapped beneath it to escape.

3. Measure the depth below the water surface of the bottom of the can, \( h \) (not the lowest point on the bottom rim of the can).

4. Remove the can from the water and dry it off. Measure the can’s diameter (use vernier calipers) and mass. Calculate the pressure at depth \( h \) from the definition of pressure (called \( P_1 \)), and the pressure-depth relation (\( P_2 \)). Check your calculations and measurements if the difference between pressures is more than ~5%!

5. Repeat this procedure using a can with a larger diameter.
Part II. Archimedes’ Principle:

Theory

If an object on a string were suspended in a beaker of water, a free body diagram would show three forces: weight $W$, (which is the same whether the object is located in air or under water), string tension $T$ and the buoyant force $F_b$. Applying Newton’s 2nd law to this static situation, we see that:

$$F_b + T - W = 0 \quad \{1\}$$

Archimedes’ Principle tells us that $F_b$ is also equal to the weight of the volume of water the object displaces. The volume of the object ($V_{block}$) is equal to the volume of the water ($V_w$) displaced. With the density of water assumed to be $\rho_w = 998.0 \text{ kg/m}^3$, you can find the mass of this volume of water ($m_w$) and hence the water’s weight ($W_w$). In this way you obtain an independent measurement of buoyant force as follows:

$$F_b = W_w = (m_w)g = (\rho_w \cdot V_w)g = (\rho_w \cdot V_{block})g \quad \{2\}$$

Experiment #2 – Archimedes’ method:

1. Use a vernier caliper to measure the dimensions of the aluminum block, recording them in a sketch; recall that the vernier calipers we use measure to 0.002 cm! Use these dimensions to calculate the volume of the block, $V_{block}$. Your final result will have 4 significant figures – don’t round before then!
   - The block has a cylindrical hole through it; how will this affect your volume calculation? If you reviewed the vernier caliper instructions, you’ll know that there is an easy way to measure the diameter of the hole!
2. You will now check your volume calculation by calculating the density of the block, and comparing this value with the published density of aluminum ($2.700 \times 10^3 \text{ kg/m}^3$). Use an electronic balance to measure the block’s mass, and calculate its density: $\rho_{block} = \frac{m_{block}}{V_{block}}$. If you measure carefully, your calculated density should be less than 2% of the published density. If not, recheck your measurements and calculations.
3. Calculate the weight of the water that will be displaced later when the block is submerged, and the buoyant force, using relation \{2\}. Remember that the volume of water displaced is equal to the volume of the block ($V_w = V_{block}$). Calculate the buoyant force to 3 significant figures.

Experiment #3 – Newton’s method:

4. Pick up a force gauge, and hold it vertically. Be sure that the gauge reads zero; ask your instructor for assistance if it needs adjustment. Attach the force gauge to the string on the block, and measure its dry weight, $W$, in Newtons.
5. Submerge the block completely under the water (at least 0.5 cm beneath the surface), making sure it doesn't hit the bottom of the container. Record the value of the tension, $T$ (estimate your reading to 0.01 N), and calculate $F_b$ using the derived force expression \{1\}.
6. Calculate the % diff between your buoyant force values. Check your measurements and calculations if the difference is more than ~5%.
7. The accuracy of the force gauge is sometimes questioned. How would you check it? Do so, and record the results in your report.
Discussion

- Restate your pressure results for the pressure-depth experiment. How well do the pressures agree with each other?
- Briefly describe the two methods used to calculate the pressure on the bottom of the cans.
- Restate the buoyant force results from the Archimedes Principle experiment. How well did they agree with each other?
- Restate Archimedes’ principle, in words. Briefly explain how it was used in this experiment.
- What are some sources of error in each experiment performed today? Be sure to also discuss your results from step 7 of experiment #3!