Double Slit Interference: Measuring the Wavelength of Light
Spring 2017

Introduction

The purpose of this experiment is to measure the wavelength of the red light from a Helium-Neon laser and to include an estimation of the uncertainty in this result. Note: There will be several lasers turned on throughout the lab; exercise caution at all times!

Theory

The following diagram defines the variables. We will assume that the wall is far enough away from the slits so that the small angle approximations are valid.

The condition for constructive interference is

\[ n\lambda = d \sin \theta_n \approx d \tan \theta_n = d \frac{x_n}{D} \]

Solving for the positions of the bright spots we get, after some rearrangement,

\[ x_n = \frac{\lambda D}{d} n \]

If we plot \( x_n \) vs. \( n \), we should get a straight line passing through the origin with a slope \( m = \frac{\lambda D}{d} \). Since we can easily measure the distance from the slits to the wall (\( D \)), and with a bit of work find the slit separation (\( d \)), it is straightforward to get the wavelength by \( \lambda = m \frac{d}{D} \).
Experiment

1. Shine a laser directly on the center pair of double-slits on the Cornell slide (shown at right) so that a good double slit interference pattern is illuminated on a piece of paper taped to the wall. Measure $D$, the distance from the Cornell slide to the wall (mark the position of the slide on the bench with tape so that the distance can be checked later).

2. With a pencil, mark the centers of the bright spots along the entire length of the paper; you may find it useful to use a ruler to mark the spot centers along a straight line. Remove the paper from the wall. Label the center spot $n = 0$, and the adjacent ones $±1, ±2$, etc. Remember that single slit diffraction puts a min on top of a double-slit interference max, so if a spot seems to be “missing”, make sure you skip a number! After numbering the spots, measure their positions, choosing $x = 0$ to be at the $n = 0$ spot.

3. Enter your data in KaleidaGraph and find the slope of the $x$ vs. $n$ line and its uncertainty, as well as the SSR. Recheck points that deviate from the best-fit line, then print the graph.

4. The slit separation can be determined using the following procedure:
   a. Place the Cornell slide in the slide projector in the orientation shown at right, and focus the image on the wall.
   b. Use a pair of vernier calipers to measure $d_{wall}$, the projected distance between the centers of the pair of slits used, as follows: Measure the distance between the top edges of the slit images; reset the caliper to zero and measure the distance between the bottom edges of the slits, as shown in the figure at right. Measure near the ends of the slits as shown at right; this will give better results if the projector is not perpendicular to the wall.

5. Calculate the average of your two measurements of $d_{wall}$. Think about how accurately you can measure this value and estimate its uncertainty, $\Delta d_{wall}$ (Throughout this experiment we will use the symbol $\Delta$ to represent uncertainty.) You can use the difference between your two measurements of $d_{wall}$ to help you estimate this uncertainty.

6. Measure $L_{wall}$ as shown in the figure of the Cornell slide above. Then measure this distance directly on the slide, $L_{slide}$.

Analysis

Calculating the wavelength, $\lambda$:

1. Once you have measured values for $d_{wall}$, $L_{wall}$ and $L_{slide}$, you can calculate the actual slit spacing on the slide, $d$ by setting up a ratio. Since the projected image on the wall is a larger scaled version of the slide, the ratio between $d$ and $L$ will always be the same, whether measured on the slide, or on the wall.

   \[
   \frac{d_{wall}}{L_{wall}} = \frac{d}{L_{slide}}
   \]

   Calculate your value of $d$, and check with your instructor that your value is reasonable before continuing.

2. Calculate the wavelength from your values of $d$ and $D$, and the slope from your Kaleidagraph plot. Convert the wavelength to units of nanometers ($\text{nm}$) { $\lambda$ should be recorded to 0.1 $\text{nm}$}. 

Calculating the uncertainty in $\lambda$:

In previous experiments, we have calculated the uncertainty in a parameter by simply using the standard error in that parameter, calculated in Kaleidagraph. In this week’s experiment, the uncertainty in the wavelength depends on the uncertainty in the slope of the best-fit line as well as the uncertainty in the slit-separation, $d$. Follow the steps below to combine to combine these uncertainties (the symbol $\Delta$ represents the uncertainty in a quantity.)

3. Estimate the uncertainty in the wavelength, $\Delta\lambda_{\text{slope}}$, due to the uncertainty in the slope of your graph, $\Delta m$. Recall that the uncertainty in the slope is twice the standard error from Kaleidagraph:

$$\Delta\lambda_{\text{slope}} = \Delta m \left( \frac{d}{D} \right)$$

4. Another source of uncertainty comes from your measurements you made to determine the slit spacing $d$. Using your estimate of the uncertainty in your measurement of $d_{\text{wall}}$, calculate the uncertainty in $d$ using the following equation:

$$\Delta d = \Delta d_{\text{wall}} \left( \frac{L_{\text{slide}}}{L_{\text{wall}}} \right)$$

5. Now you can find the uncertainty in the wavelength due to the uncertainty in the slit separation:

$$\Delta\lambda_{\text{slit}} = \Delta d \left( \frac{m}{D} \right)$$

6. Which uncertainty is larger: the one due to the slope of the graph, or the one due to the slit spacing? You can combine them by taking the square root of the sum of the squares.

$$\Delta\lambda = \sqrt{\left(\Delta\lambda_{\text{slope}}\right)^2 + \left(\Delta\lambda_{\text{slit}}\right)^2}$$

7. Compare your calculated wavelength to the actual value and see if your result is consistent within the measurement uncertainty.

Discussion

- Restate your calculated value and uncertainty of $\lambda$, and discuss the source of errors involved in its calculation.
- Did the actual value of $\lambda$ fall within the calculated range of your measurement uncertainty?
- Which measured value is the most critical in this experiment? Why?

**WHEN FINISHED, PLEASE TURN OFF THE FLASHLIGHT AND LASER, AND REMOVE ANY TAPE APPLIED TO THE LAB BENCH!**