HOMEWORK SET 2: 1-D SCHRÖDINGER REVIEW
Due Wednesday, January 22, 2025

QUANTUM MECHANICAL TOOLS TO REMEMBER:

Wave Function

e.g. Particle in a 1-D box: $\psi(x) = \sqrt{\frac{2}{a}}\sin\left(\frac{n\pi x}{a}\right)$ TZDII (7.60)

Probabilty Density (probability/unit length in 1-D)

$|\psi(x)|^2$ = probability (linear) density for finding particle at x

Prob. of finding particle between x_1 & x_2 = $\int_{x_1}^{x_2} |\psi(x)|^2\,dx \approx |\psi(x = x_1)|^2\,\Delta x$ TZDII (6.15) & (7.63)

WHY IS INTEGRAL APPROXIMATED BY A PRODUCT?

Normalization (requiring probability of particle existing on x is 1 … it does exist somewhere!

$$\int_{-\infty}^{\infty} |\psi(x)|^2 dx = 1$$ TZDII (7.55)

CARTESIAN COORDINATES GO FROM -∞ TO +∞

Expectation Value (value expected after many measurements)

For a function (e.g. position) f(x) with a probability density $|\psi(x)|^2$ is

$$\int f(x)|\psi(x)|^2\,dx = \int f(x)p(x)dx$$ TZDII (7.69)

PROBLEMS FROM TZDII[1]

1) 7.13 A general (real) wave has time dependenc written as

$$\psi(t) = a\cos(\omega t) + b\sin(\omega t) \quad \text{or} \quad \psi(t) = A\sin(\omega t + \phi)$$

a) Show that the two forms are equivalent.

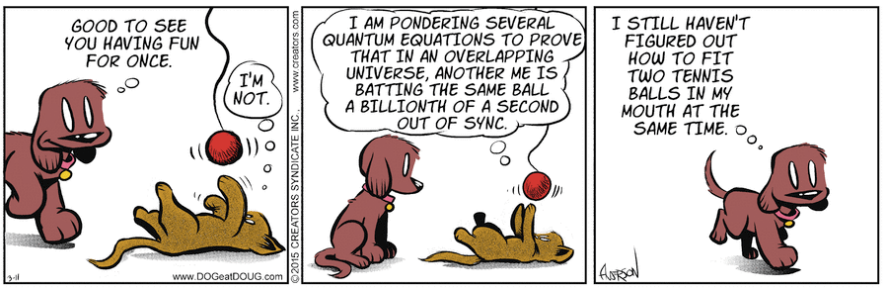
b) Show that chaging the origin of time can eliminate ϕ.

2) 7.24 a) Prove that the function $\psi = Ae^{ikx} + Be^{-ikx}$ satisfies the equation $\psi'' = -k^2\psi$ for any constants A and B.

3) 7.31 a) Write down and sketch the probability distribution of $|\psi(x)|^2$ for the third excited state (n = 4) of a particle in a rigid box of lenth a.

b) What are the most probable positions, x_{mp}?

c) What are the probablilies of finding the particles in the intervals [0.50a,0.51a] and [0.75a,0.76a]. USE THE APPROXIMATION ABOVE .



[1] Taylor, Zafiratos, & Dubson, *Modern Physics for Scientists and Engineers*, 2nd Editon, Pearson, Prentice Hall, 2004