HOMEWORK SET 3: PARTIAL DERIVATIVES
Due Friday, January 24, 2025

Example: Find the partial derivatives of f (x, y, z) = z sin(x – y).

$$\frac{\partial f}{\partial x} = z\cos(x-y),\ \frac{\partial f}{\partial y} = -z\cos(x-y),\ \frac{\partial f}{\partial z} = \sin(x-y)$$

PROBLEMS FROM AOD

1) Find the partial derivatives of the following functions:

a) $f(x, y) = 3x^2 - xy + y$

b) $f(x, y) = e^{x-y} - x^2y$

c) $\rho(\theta, \phi) = (\theta - \frac{1}{2}\pi)\sin(\phi - \frac{1}{2}\pi)$

d) $\rho(\theta, \phi) = e^{\theta+\phi}\cos(\theta - \phi)$

e) Show that $u = Ax^4 + 2Bx^2y^2 + Cy^4$ satisfies the equation $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 4u$

f) Show that $u = \frac{x^2y^2}{x+y}$ satisfies the equation $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 3u$

PROBLEMS FROM TZDII[1]

1) 8.18 Changes of coordinates in two dimensions (such as that from x, y to r, ϕ) are much more complicated than in one dimension. in one dimension, if we have a function f(x) and choose to regard x as a fundtion of some other variable u, then the derivative of f with respect to u is given by the chain rule,

$$\frac{df}{du} = \frac{df}{dx}\frac{dx}{du}$$

In 2 dimensions the chain rule reads

$$\frac{\partial f}{\partial r} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial r} \qquad \text{TZDII (8.105)}$$

and

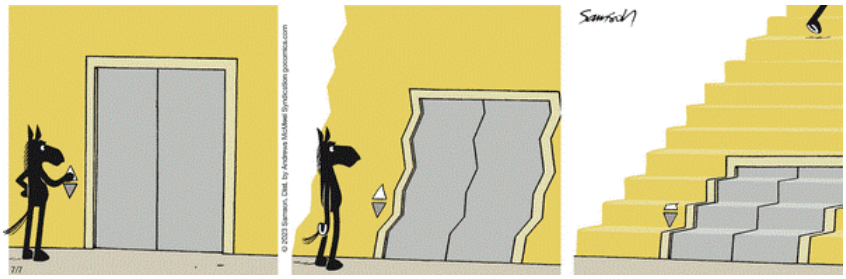
$$\frac{\partial f}{\partial \phi} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial \phi} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial \phi}$$

a) Use the relations $x = r\cos(\phi)$ and $y = r\sin(\phi)$ [TZDII (8.103)] to evaluate the four derivatives .

b) If $f = e^{\sqrt{x^2+y^2}}$, use (8.105) to find $\partial f/\partial r$.

c) What is $\partial f/\partial \phi$?

d) By noticing that $\sqrt{x^2+y^2} = r$ and hence that $f = e^r$, evaluate $\partial f/\partial r$ and $\partial f/\partial \phi$ directly and check that your answers in parts (b) and (c) are correct.



[1] Taylor, Zafiratos, & Dubson, *Modern Physics for Scientists and Engineers*, 2nd Editon, Pearson, Prentice Hall, 2004