HOMEWORK SET 05: SEPARATION OF VARIABLES
Due Monday, February 3, 2025

PROBLEMS FROM AOD

1) Show that

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right)\psi = \frac{1}{r}\frac{\partial^2}{\partial r^2}(r\psi)$$

PROBLEMS FROM TZDII[1]

8.22) Paraphrased

$$\frac{1}{r}\frac{\partial^2}{\partial r^2}(r\psi) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right)\psi + \frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\phi^2}\psi = -\frac{2M}{\hbar^2}\left[U(r)-E\right]\psi \qquad (8.49)$$

a) Substitute $\psi = R\Theta\Phi$, then multiply by $r^2\sin^2\theta/R\Theta\Phi$ recognizing that partial derivatives only act on a specific variable and functions of any other variable are constants.

b) Keep the functions of ϕ on the left hand side and functions of r and θ on the right hand side. Explain clearly why each side of this equation must be a constant which we will set as $-m^2$. Note that the partial derivative becomes a standard derivative in a single-variable equation to derive

$$\frac{d^2}{d\phi^2}\Phi = -m^2\Phi \qquad (8.51)$$

c) From the right hand side above, separate the functions of r & θ. Explain (again) why each side of this equation must be a constant that we will call –k to derive

$$\frac{1}{\sin\theta}\frac{d}{d\theta}\left(\sin\theta\frac{d\Theta}{d\theta}\right) + \left(k - \frac{m^2}{\sin^2\theta}\right)\Theta = 0 \qquad (8.53)$$

and

$$\frac{d^2}{dr^2}(rR) = \frac{2M}{\hbar^2}\left[U(r) + \frac{k\hbar^2}{2mr^2} - E\right](rR) \qquad (8.54)$$

These are the one-dimensional differential equations resulting from the separation of variables that we can solve individually.

8.25) a) Draw a (correctly scaled … use graph paper and your ruler) vector model diagram similar to Fig 8.14 for angular momentum of magnitude given by ℓ = 3.

b) How many possible orientations are there?

c) What is the minimum angle between $\vec{L}$ and the z axis? (USE THE RATIO OF $L_z/|\vec{L}|$ TO GET AN EXACT ANGLE (USE THE TRIG CHEAT SHEET), DON'T USE YOUR CALCULATOR!)

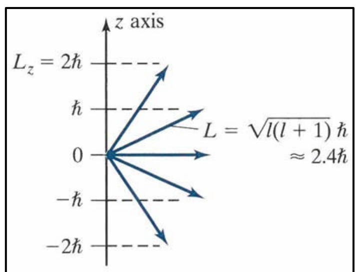
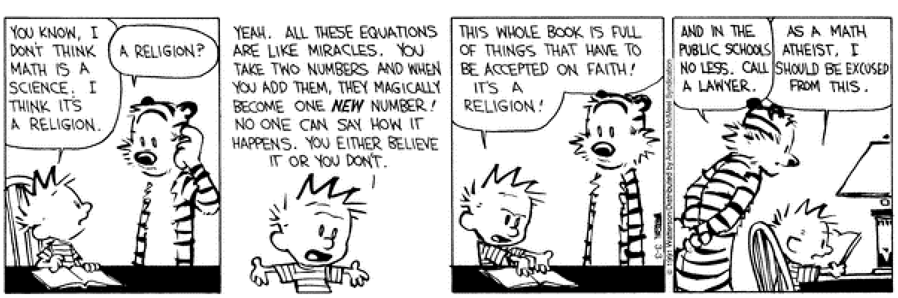


FIGURE 8.14
Classical representation of the quantized values of angular momentum **L** for the case $l = 2$. The z component has $(2l + 1) = 5$ possible values, $L_z = m\hbar$ with $m = 2, 1, 0, -1, -2$. The magnitude of **L** is $L = \sqrt{l(l+1)}\hbar = \sqrt{2 \times 3}\hbar \approx 2.4\hbar$ in all five cases.



[1] Taylor, Zafiratos, & Dubson, *Modern Physics for Scientists and Engineers*, 2nd Editon, Pearson, Prentice Hall, 2004