Homework Set 06: Theta and R Equations
Due Friday, February 7, 2025

Problem adapted from TZDII[1]

8.30) a) Write down the complete θ equation for the case $\ell = 2$, m = 1

$$\frac{1}{\sin\theta}\frac{d}{d\theta}\left(\sin\theta\frac{d\Theta_{\ell,m}}{d\theta}\right)+\left(\ell\left(\ell+1\right)-\frac{m^2}{\sin^2\theta}\right)\Theta_{\ell,m}=0 \qquad \text{TZDII (8.65)}$$

b) Show that the $\Theta_{2,1}(\theta)$ solution below does satisfy this equation (TAKE THE DERIVATIVE OF $\Theta_{\ell,M}(\Theta)$, SUBSTITUTE AND TAKE THE DERIVATIVE OS SIN(Θ) TIME THIS AND DO THE ALGEBRA TO GET 0 = 0)

	$\ell = 0$	$\ell = 1$	$\ell = 2$
m = 0	$\Theta_{0,0}=\sqrt{\frac{1}{4\pi}}$	$\Theta_{1,0}=\sqrt{\frac{3}{4\pi}}\cos\theta$	$\Theta_{2,0}=\sqrt{\frac{5}{16\pi}}\left(3\cos^2\theta-1\right)$
m = ± 1		$\Theta_{1,\pm1}=\mp\sqrt{\frac{3}{8\pi}}\sin\theta$	$\Theta_{2,\pm1}=\mp\sqrt{\frac{15}{8\pi}}\sin\theta\cos\theta$
m = ± 2			$\Theta_{2,\pm2}=\sqrt{\frac{15}{32\pi}}\sin^2\theta$

8.39) a) Verify that

$$\frac{d}{dr^2}\left(rR\right)=\frac{2m_e}{\hbar^2}\left(-\frac{ke^2}{r}+\frac{E_R}{n^2}\right)\left(rR\right) \qquad \text{TZDII (8.79)}$$

can be written as

$$\frac{d}{dr^2}\left(rR\right)=\left(\frac{1}{n^2a_B^2}-\frac{2}{a_Br}\right)\left(rR\right) \qquad \text{TZDII (8.107)}$$

Where $a_B=\frac{\hbar^2}{m_eke^2}$ and $E_R=\frac{ke^2}{2a_B}$.

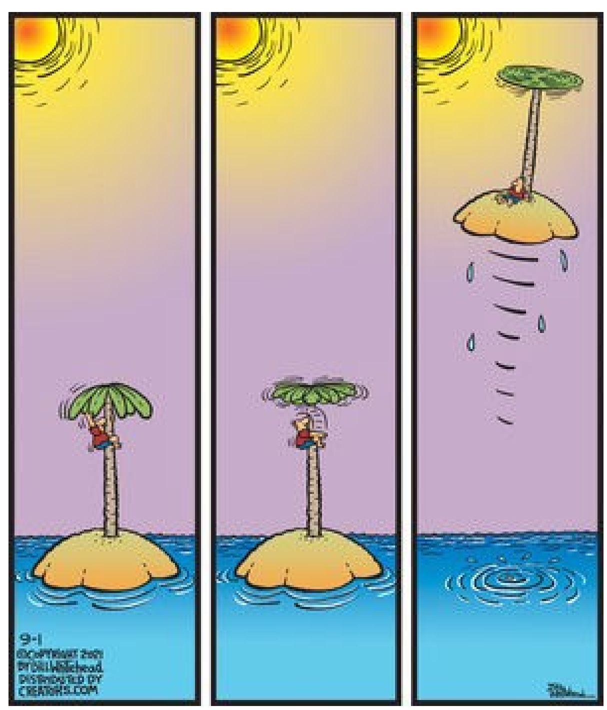
b) for the case that n = 1, prove that $R_{1s}\left(r\right)=e^{-r/a_B}$ is a solution of (8.107).

8.44) a) Verify that

$$R_{2,0}\left(r\right)=A\left(1-\frac{r}{2a_B}\right)e^{-r/2a_B}$$

Satisfies the radial Schrödiger Equation (8.107)

b) Find A from normalization.



[1] Taylor, Zafiratos, & Dubson, *Modern Physics for Scientists and Engineers*, 2nd Editon, Pearson, Prentice Hall, 2004