2) SHOW $x = Ae^{-\beta t}\cos(\omega t)$ SOLVES

$$\underbrace{\frac{d^2x}{dt^2} + 2\beta\frac{dx}{dt} + \omega^2 x = 0}_{n}$$

START BY TAKING DERIVATIVES

$$\frac{d}{dt}Ae^{-\beta t}\cos(\omega t) = Ae^{-\beta t}\left[-\beta\cos(\omega t) - \omega\sin(\omega t)\right]$$

$$\frac{d^2}{dt^2}Ae^{-\beta t}\cos(\omega t) = Ae^{-\beta t}\Big\{-\beta\left[-\beta\cos(\omega t) - \omega\sin(\omega t)\right] +$$

$$+\left[\beta\omega\sin(\omega t) - \omega^2\cos(\omega t)\right]\Big\}$$

$$\frac{d^2}{dt^2}Ae^{-\beta t}\cos(\omega t) = Ae^{-\beta t}\left[(\beta^2-\omega^2)\cos(\omega t) + 2\beta\omega\sin(\omega t)\right]$$

NOW SUBSTITUTE INTO THE DE:

$$\frac{d^2x}{dt^2} + 2\beta\frac{dx}{dt} + \omega^2 x = 0$$

$$Ae^{-\beta t}\Big\{\left[(\beta^2-\omega^2)\cos(\omega t) + 2\beta\omega\sin(\omega t)\right] +$$

$$2\beta\left[-\beta\cos(\omega t) - \omega\sin(\omega t)\right] + \omega^2\cos(\omega t)\Big\} = 0$$

NOTE $Ae^{-\beta t}$ CANCELS OUT & GATHER COEFFICIENTS OF $\cos(\omega t)$ AND $\sin(\omega t)$:

$$\left[\beta^2 - \omega^2 - 2\beta^2 + \omega^2\right]\cos(\omega t) + \left[\cancel{2\beta\omega - 2\beta\omega}\right]\sin(\omega t) = 0 \quad \text{ZERO}$$

$$\left[\beta^2 - 2\beta^2 \underbrace{\cancel{-\omega^2} + \cancel{\omega^2}}_{\text{CANCEL}}\right]\cos(\omega t) = 0$$

$$-\beta\cos(\omega t) \neq 0!$$

BECAUSE AILEEN OVER SIMPLIFIED THE DE!

$$\ddot{x} + 2\beta\dot{x} + \omega_N^2 x = 0, \quad x = Ae^{-\beta t}\cos(\omega_S t)$$

$$\omega_S^2 = \omega_N^2 - \beta^2. \quad \text{Duh!}$$