HOMEWORK SET 2: 1-D SCHRÖDINGER REVIEW
Phase constants in trig. functions

PROBLEMS FROM TZDII[1]

1) 7.13 A general (real) wave has time dependence written as

$$\psi(t) = a\cos(\omega t) + b\sin(\omega t) \quad \text{or} \quad \psi(t) = A\sin(\omega t + \phi)$$

a) Show that the two forms are equivalent.

b) Show that changing the origin of time can eliminate ϕ.

a) Using the angle sum identity for sine,

$$\psi(t) = A\sin(\omega t + \phi) = A\left[\sin(\omega t)\cos(\phi) + \cos(\omega t)\sin(\phi)\right]$$

Since ϕ is just a constant, both $\cos(\phi)$ and $\sin(\phi)$ are constants so rename them as $b = \cos(\phi)$ and $a = \sin(\phi)$, then

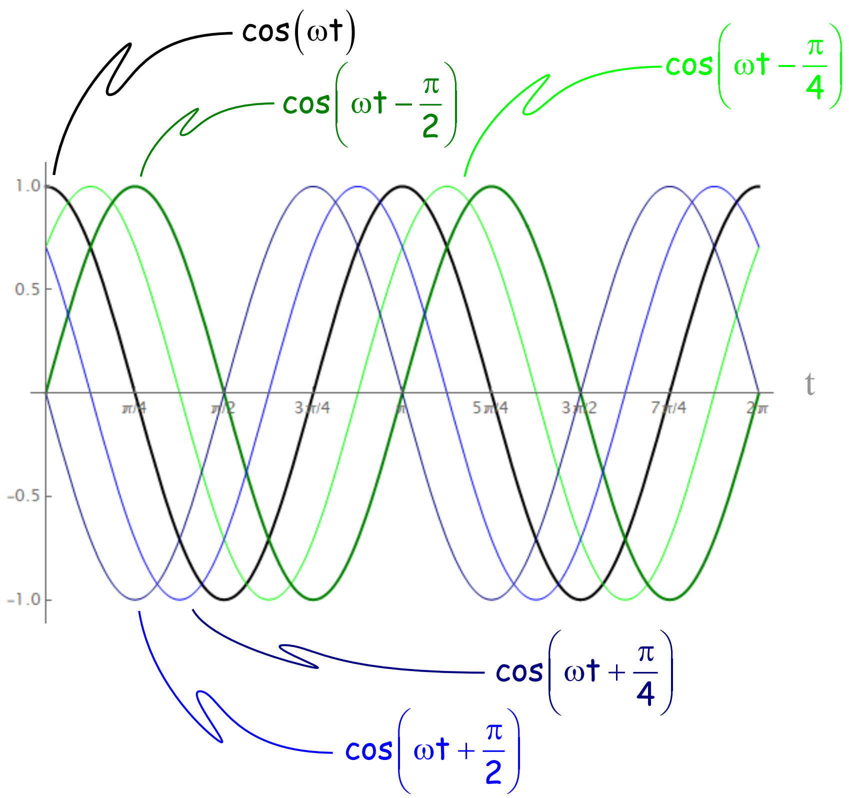
$$\psi(t) = A\left[\sin(\omega t)\cos(\phi) + \cos(\omega t)\sin(\phi)\right] = A\left[b\sin(\omega t) + a\cos(\omega t)\right]$$

b) Show that changing the origin of time can eliminate ϕ.

Since ϕ is just a constant, define t' to eliminate ϕ:

$$\omega(t') = (\omega t + \phi)$$

This just serves to shift the t = 0 axis to a new value as shown in these Mathematica plots:



The green curves show that subtracting a phase constant moves the t = 0 "backwards" relative to the original cosine curve, whereas the blue curves show that subtracting the phase constant moves the t = 0 curves "forward" relative to the original cosine curve. Note that

$$\cos\left(\omega t - \frac{\pi}{2}\right) = \sin(\omega t)$$

and, consequently

$$\sin\left(\omega t - \frac{\pi}{2}\right) = \cos(\omega t)$$

[1] Taylor, Zafiratos, & Dubson, *Modern Physics for Scientists and Engineers*, 2nd Editon, Pearson, Prentice Hall, 2004

Plots of the sine function with and without a phase constant. Now subtracting the phase constant moves the t = 0 axis forward along the sine function.