1) FIND THE PARTIAL DERIVATIVE

a) $f(x,y) = 3x^2 - xy + y$

$$\frac{\partial f}{\partial x} = 6x - y \qquad \frac{\partial f}{\partial y} = x + 1$$

b) $f(x,y) = e^{x-y} - x^2 y$

$$\frac{\partial f}{\partial x} = e^{x-y} - 2xy \qquad \frac{\partial f}{\partial y} = -e^{x-y} - x^2$$

c) $\rho(\theta,\phi) = (\theta - \frac{1}{2}\pi)\sin(\phi - \frac{1}{2}\pi)$

$$\frac{\partial \rho}{\partial \theta} = \sin(\phi - \tfrac{1}{2}\pi) \qquad \frac{\partial f}{\partial \phi} = (\theta - \tfrac{1}{2}\pi)\cos(\phi - \tfrac{1}{2}\pi)$$

d) $\rho(\theta,\phi) = e^{\theta+\phi}\cos(\theta - \phi)$

$$\frac{\partial \rho}{\partial \theta} = e^{\theta+\phi}\left[\cos(\theta-\phi) - \sin(\theta-\phi)\right]$$

$$\frac{\partial \rho}{\partial \phi} = e^{\theta+\phi}\left[\cos(\theta-\phi) + \sin(\theta-\phi)\right]$$

e) $u = Ax^4 + 2\beta x^2 y^2 + Cy^4$ SOLVES $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 4u$

$$\frac{\partial u}{\partial x} = 4Ax^3 + 4\beta x y^2 \qquad \frac{\partial u}{\partial y} = 4\beta x^2 y + 4Cy^3$$

SUB INTO DE: $x\left[4Ax^3 + 4\beta x y^2\right] + y\left[4\beta x^2 y + 4Cy^3\right] = 4u$

$$4\left(Ax^4 + \beta x^2 y^2 + \beta x^2 y^2 + Cy^4\right) = 4u$$

$$4\left(Ax^4 + 2\beta x^2 y^2 + Cy^4\right) = 4u \quad \text{QED!}$$

f →

1) CONTINUED

f) $u = \dfrac{x^2 y^2}{x+y}$ SOLVES $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = 3u$

$$\frac{\partial u}{\partial x} = (2xy^2)(x+y)^{-1} - x^2y^2(x+y)^{-2}(1) = \frac{2xy^2(x+y) - x^2y^2}{(x+y)^2}$$

$$\frac{\partial u}{\partial y} = (2x^2y)(x+y)^{-1} - x^2y^2(x+y)^{-2}(1) = \frac{2x^2y(x+y) - x^2y^2}{(x+y)^2}$$

SUBSTITUTE INTO THE DE

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 3u$$

$$\frac{2x^2y^2(x+y) - x^3y^2}{(x+y)^2} + \frac{2x^2y^2(x+y) - x^2y^3}{(x+y)^2} \overset{?}{=} 3u$$

$$\frac{2(2x^2y^2)\cancel{(x+y)} - x^2y^2\cancel{(x+y)}}{(x+y)^{\cancel{2}}} \overset{?}{=} 3u$$

CANCELING THE COMMON $(x+y)$ FACTORS GIVES

$$\frac{4x^2y^2 - x^2y^2}{(x+y)} \overset{?}{=} 3u$$

$$\frac{3x^2y^2}{(x+y)} \overset{?}{=} 3u$$

$$3\left(\frac{x^2y^2}{x+y}\right) \overset{?}{=} 3u$$

$$3u = 3u \quad \text{QED!}$$