

8.18) a) USE $x = r \cos \phi$ AND $y = r \sin \phi$ (8.104) TO EVALUATE

$$\frac{\partial x}{\partial r}, \frac{\partial y}{\partial r}, \frac{\partial x}{\partial \phi} \text{ AND } \frac{\partial y}{\partial \phi}$$

b) IF $f = e^{\sqrt{x^2+y^2}}$, USE $\frac{\partial f}{\partial r} = \frac{\partial f}{\partial x} \frac{\partial x}{\partial r} + \frac{\partial f}{\partial y} \frac{\partial y}{\partial r}$ TO FIND $\frac{\partial f}{\partial r}$.

c) WHAT IS $\frac{\partial f}{\partial \phi}$?

d) BY NOTICING $r = \sqrt{x^2+y^2}$ AND $f = e^r$, EVALUATE $\frac{\partial f}{\partial r}$ AND $\frac{\partial f}{\partial \phi}$ DIRECTLY AND CHECK YOUR ANSWERS IN b & c.

$$a) \frac{\partial x}{\partial r} = \cos \phi, \quad \frac{\partial x}{\partial \phi} = -r \sin \phi$$

$$\frac{\partial y}{\partial r} = \sin \phi, \quad \frac{\partial y}{\partial \phi} = r \cos \phi$$

$$b) \frac{\partial f}{\partial r} = \frac{\partial f}{\partial x} \frac{\partial x}{\partial r} + \frac{\partial f}{\partial y} \frac{\partial y}{\partial r}, \text{ SO FOR } f = e^{\sqrt{x^2+y^2}}$$

$$\frac{\partial f}{\partial r} = \frac{1}{\sqrt{x^2+y^2}} \left[x(\cos \phi) + y(\sin \phi) \right] e^{\sqrt{x^2+y^2}}$$

$$\frac{\partial f}{\partial r} = \frac{x \cos \phi + y \sin \phi}{\sqrt{x^2+y^2}} e^{\sqrt{x^2+y^2}}$$

REPLACING $x = r \cos \phi$, $y = r \sin \phi$ AND $r = \sqrt{x^2+y^2}$ GIVES

$$\frac{\partial f}{\partial r} = \frac{r \cos^2 \phi + r \sin^2 \phi}{r} e^r$$

$$\Rightarrow \boxed{\frac{\partial f}{\partial r} = e^r}$$

$$c) \frac{\partial f}{\partial \phi} = \frac{\partial f}{\partial x} \frac{\partial x}{\partial \phi} + \frac{\partial f}{\partial y} \frac{\partial y}{\partial \phi}$$

$$\frac{\partial f}{\partial \phi} = \frac{1}{\sqrt{x^2+y^2}} \left[x(-r \sin \phi) + y(r \cos \phi) \right] e^{\sqrt{x^2+y^2}}$$

NOTING $r \sin \phi = y$ AND $r \cos \phi = x$

$$\frac{\partial f}{\partial \phi} = \frac{1}{\sqrt{x^2+y^2}} \left[-xy + xy \right] e^{\sqrt{x^2+y^2}} = \boxed{0 = \frac{\partial f}{\partial \phi}}$$

d) For $f = e^r$

$$\frac{\partial f}{\partial r} = e^r \quad \text{AND} \quad \frac{\partial f}{\partial \phi} = 0 \Rightarrow \text{b \& c ARE CORRECT}$$