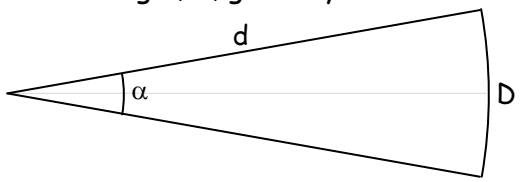
Intensity to Surface Brightness in Magnitudes

For galaxies, we measure a surface flux, that is, the counts in each pixel. Through calibration, this is converted to flux density in Janskys (1 Jy = 10^{-26} W/m^2/Hz).

For a galaxy[1] at some distance, d, a pixel of side D subtends an angle, α, given by

$$\alpha = \frac{D}{d}.$$



The surface brightness is the amount of light in that patch of sky divided by its area (in arsec^2):

$$I(r) = \frac{\text{Flux}}{\alpha^2} \sim \frac{\text{Jy}}{\text{arcsec}^2}.$$

Recalling the relationship between flux and luminosity, $\text{Flux} = \frac{\text{Luminosity}}{4\pi d^2}$, the surface brightness becomes

$$I(r) = \frac{F}{\alpha^2} = \frac{L}{4\pi d^2}\left(\frac{d}{D}\right)^2 = \frac{L}{4\pi D^2} \sim \frac{L_\odot}{pc^2} \text{ or } \frac{\text{Watts}}{\text{arcsec}^2}.$$

Which is often given in solar luminosities per parsec^2.

To convert this to magnitudes, recall that the apparent magnitude is a measure of flux,

$$m - m_0 = -2.5 \log\left(\frac{F}{F_0}\right).$$

So the surface brightness in magnitudes per arsec^2 is

$$\mu - \mu_0 = -2.5 \log\left(\frac{F/\alpha^2}{F_0/\alpha^2}\right),$$

$$\mu - \mu_0 = -2.5 \log\left(\frac{I}{I_0}\right).$$

EXAMPLE: For the Sun in the optical V-Band, taking $I_0 = 1\ L_\odot/pc^2$ for $\mu_0 = 26.4$ magnitudes/arcsec^2, yields

$$\mu = -2.5 \log(I) + 26.4 \sim \text{mag/arcsec}^2.$$

Giving the surface brightness (in the V-band in this case), as

$$I_V = 10^{0.4(26.4-\mu_V)} \frac{L_{\odot V}}{pc^2}.$$

[1] From Amina Helmi's Kapteyn course at www.astro.rug.nl/~ahelmi/teaching/gal2010/ellipt.pdf